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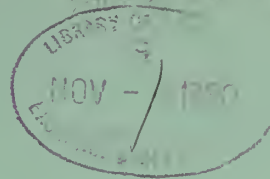
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Bureau of Mines Statistical Projection Methodology of U. S. Mineral Consumption by End Use: Aluminum as an Example

By William Y. Mo and Barry W. Klein



UNITED STATES DEPARTMENT OF THE INTERIOR

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BUREAU OF MINES STATISTICAL PROJECTION METHODOLOGY OF U.S. MINERAL CONSUMPTION BY END USE: ALUMINUM AS AN EXAMPLE

by

William Y. Mo¹ and Barry W. Klein¹

ABSTRACT

This Information Circular provides a detailed background documentation of how the Bureau of Mines projects U.S. mineral demand to the year 2000. These statistical projections serve as a quantitative basis for contingency forecasting of the low, high, and probable U.S. demand for nonfuel minerals by end-use categories and are published in the Bureau of Mines Mineral Commodity Profiles and Mineral Facts and Problems series. Aluminum is used as an example in this study.

INTRODUCTION

The Bureau of Mines forecasting system consists of two important components: (1) statistical analysis and (2) contingency analysis. The system is currently used in deriving long-range U.S. demand forecasts for mineral commodities by end-use categories to the year 2000. The statistical analysis is performed by the analytical staff of the Division of Analytic Studies, and the contingency analysis is carried out by commodity specialists of the Division of Production/Consumption Data Collection and Interpretation. The statistical analyses are based on a standard least-squares procedure, while the contingency analyses are of a qualitative nature based on the commodity specialists' knowledge and judgment. The results of statistical analysis are referred to as "statistical projections," and the results of contingency analysis are called "contingency forecasts." These demand projections and forecasts by end use are published in the Bureau of Mines Mineral Commodity Profiles and Mineral Facts and Problems series to provide the general public and decisionmakers with an overview of present and probable future supply-demand relationships for individual mineral industries.

Although the statistical analysis method used in the Bureau of Mines statistical demand projection system is the ordinary least-squares, or regression analysis, method, it is not familiar to many users of Bureau demand projections and forecasts. Accordingly, this Information Circular provides documentation for the methodology and procedure for deriving these projections, which are made for all nonfuel minerals, using aluminum as an example. The

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main part of the publication is simplified to the extent possible, and a more rigorous mathematical derivation is presented in an appendix.

GENERAL METHOD

Changes are taking place constantly in a modern society. Decisionmakers, in both private and government sectors, not only are faced with present problems but also are required to anticipate future events. As a result, projection and forecasting activities have become an indispensable part of an informed decisionmaking process. Of course, anticipating future occurrences has always been a very difficult task. To deal with this problem, many statistical methods, both simple and complicated, have been developed and applied. Among all the methods that have been developed, regression analysis is one of the most frequently used techniques. The Bureau of Mines uses this simple technique to derive a set of statistical mineral demand projections by end use to the year 2000.

In the Bureau of Mines statistical demand projection system, it is assumed that the end-use consumption of a mineral commodity can be approximated by a simple linear regression equation as follows:

$$Y_t = a + b X_t + e_t \quad (\text{for } t = 1, 2, 3, \dots, n) \quad (1)$$

where Y_t = end-use consumption of a mineral commodity at time t , X_t = a macroeconomic variable at time t , e_t = a disturbance term, and a and b are parameters.

Equation 1 specifies that end-use consumption of a mineral commodity is related primarily with a macroeconomic variable. Because mineral commodities are basic raw materials used in our economy, it is not unreasonable to expect that the demand for a mineral commodity will depend upon general economic conditions, and that the demand for the mineral will change as economic conditions change. Furthermore, mineral commodities may be used by different sectors of the economy in a variety of ways. Therefore, 38 macroeconomic variables are selected as possible explanatory variables. A list of these variables is given in appendix A.

Equation 1 is defined basically by two parameters, a and b , but the relationship is inexact because it also contains a disturbance (or error) term e_t . The inclusion of an error term in equation 1 is to take into account the influence of omitted variables, the error of approximation of the functional form, and other unpredictable random effects.

The unknown parameters, a and b , must be estimated by reference to observable data for some particular historical period. The mathematical derivations of the estimation formulas are quite complex (appendix B). Basically, the method attempts to find numerical estimates for the parameters a and b that, when used in equation 1, best explain the known historical data. In other words, the objective of the method is to estimate parameter values that make the accumulated squares of the sample-period prediction errors as small as possible. This is commonly known as the least-squares criterion.

Let $\hat{a}$ and $\hat{b}$ represent the least-squares estimated values of a and b . Then, as indicated in appendix B, the $\hat{a}$ and $\hat{b}$ can be obtained by using the following formulas:

$$\hat{b} = \frac{\sum_{t=1}^n X_t Y_t - n\bar{X}\bar{Y}}{\sum_{t=1}^n X_t^2 - n\bar{X}^2} \quad (2)$$

$$\hat{a} = \bar{Y} - \hat{b}\bar{X} \quad (3)$$

where $\bar{X} = (1/n) \sum_{t=1}^n X_t$, and $\bar{Y} = (1/n) \sum_{t=1}^n Y_t$.

Furthermore, with additional assumptions about the disturbance term e_t in equation 1, the above least-squares estimators have additional desirable properties such as the "best linear unbiased" and "maximum likelihood" features. The detailed explanations are given in appendix B.

After several estimated equations have been obtained for an end-use consumption of a mineral commodity, it becomes necessary to determine which estimated equation should be selected as a basis for deriving the statistical projection. Obviously, among the estimated equations, the equation that has the smallest sum of the squared residuals (or predicted errors) should be selected. From the following definition of coefficient of determination (commonly known as R^2), it is clear that this is equivalent to selecting the estimated equation that has the highest R^2 value.

$$R^2 = 1 - \frac{\sum_{t=1}^n e_t^2}{\sum_{t=1}^n (Y_t - \bar{Y})^2} \quad (4)$$

where $\sum_{t=1}^n e_t^2 = \sum_{t=1}^n [Y_t - \hat{Y}_t]^2 = \sum_{t=1}^n [Y_t - \hat{a} - \hat{b} X_t]^2$

= sum of squared residuals.

From equation 4 it is clear that the maximum value of R^2 must be unity, and this can only occur when $\sum_{t=1}^n e_t^2 = 0$; that is, when all historical data

points lie exactly on the estimated regression line; the observed values of the dependent variable Y_t would be equal to the predicted values $\hat{Y}_t$ calculated from the estimated regression line. As the independent (or explanatory) variable explains less and less of the variation in the dependent variable, the value of R^2 falls closer and closer toward zero. Hence, the R^2 value provides a

useful measure that can be used to determine which macroeconomic variable best describes the historical pattern of each end-use consumption and, therefore, which macroeconomic variable should be used as an explanatory factor for deriving each end-use statistical projection. Furthermore, if we assume that the relationship shown by the selected estimated equation will continue into the future, then projections can be easily obtained by solving this estimated equation for the variable to be projected by substituting the appropriate period's value for the macroeconomic explanatory variable in the estimated equation.

STATISTICAL DEMAND PROJECTION PROCEDURE

In the preceding section, the general method used in the Bureau of Mines statistical demand projection system was described briefly. The procedures for actual implementation of this methodology are explained in this section.

To increase the implementation efficiency, the Branch of Economic Analysis developed a computerized statistical demand projection system. However, it is worth bearing in mind that a computer does not "make projections," but rather carries out the tedious routine calculations that serve as the basis for deriving the projections. This computerized system performs statistical regression analyses based on the historical data of each mineral commodity's end-use consumption, and produces graphs of the historical relationships between U.S. mineral commodity end-use consumption and various macroeconomic explanatory variables. It also estimates the underlying regression equations and the associated coefficients of determination. These results provide information as to which macroeconomic variable best explains the historical pattern of end-use consumption, and consequently which estimated relationship should be used as a basis for deriving end-use consumption projections. The implementation of this system involves the following operational procedures.

Compilation of Data

The first step in this procedure is to compile the dependent-variable data, which are annual time series data of end-use consumptions. The recent estimated equations were based on data from 1960 through 1977. Portions of these data are published in the Bureau of Mines publication "Minerals in the U.S. Economy: Ten-Year Supply-Demand Profiles for Nonfuel Mineral Commodities (1968-77)." In some cases, the estimated equations were based on, for example, only 13 and 14 years of data, because the end-use consumption data were not available as far back as 1960. In order to improve the usefulness of the data collected, the Bureau of Mines has recently developed canvasses for selected commodities, requesting end-use data according to the Standard Industrial Classification (SIC) breakdown.

The U.S. annual data for the independent variables, shown in appendix A, are compiled from various Government sources. The U.S. annual data on gross national product (GNP), population, gross private domestic investment (GPDI), and new construction activity are all from the Department of Commerce, and industrial production indexes are obtained from the Federal Reserve

Board (FRB). The data from 1960 through 1977 for these explanatory variables are stored on a computer tape for easy retrieval.

Selection of Independent Variables

Although the Bureau of Mines presently maintains 38 macroeconomic variables as possible explanatory variables, only 4 of them (GNP, population, GPDI, and the FRB total industrial production index) are currently used for estimating end-use consumption equations for every commodity. In the earlier analyses, before improvements were made in the procedure, not all four explanatory variables were used in some cases. These variables are now used in every case because they are more general economic indicators than the remaining industry-specific variables. To determine which of the remaining explanatory variables are logically related to any given end use of a commodity, the appropriate commodity specialist is consulted. Therefore, there are a minimum of four, and usually more, estimated linear equations for any given end use of a commodity.

Selection of Estimated Equation Used for Projection

To select the estimated equation used for deriving an end-use projection for any given commodity, the R^2 criterion is used, as explained in the "General Method" section. To provide users with a clear idea of the goodness of fit of the underlying estimated equations, a decision has been made that a statistical projection derived from an estimated equation with an R^2 value greater than or equal to 0.70 should be distinguished from one with an R^2 value below 0.70. This distinction has been indicated in the projections and forecasts table in each Mineral Commodity Profile since early 1978 and will be made in the 1980 edition of Mineral Facts and Problems.

Determination of End-Use Projection

To use the estimated regression equation for making projections, the year 2000 value of the explanatory variable in each regression equation has to be estimated or assumed. Values of these macroeconomic variables were obtained from several sources. The U.S. population value in year 2000 was based on a Department of Commerce, Bureau of the Census series. A set of consistent estimated values for U.S. GNP and FRB industrial production indexes, from 1976 through 1990, was provided by Data Resources, Inc. (DRI)² from its macroeconomic model. Based on this information, the values for year 2000 were estimated. From these assumed values and the estimated relationships, the end-use projections were obtained by the technique described in the "General Method" section. The Bureau is currently having these estimated values updated.

It should be noted that in a few cases a particular end use for a commodity has a rapidly declining historical pattern. Consequently, the projection will become zero in, or even before, the year 2000. In these cases we assume the year 2000 end-use consumption will be zero.

²DRI, Lexington, Mass., is one of several consulting firms that provide macroeconomic forecasting models to clients on a subscriber fee basis.

ALUMINUM AS AN EXAMPLE OF THE STATISTICAL DEMAND PROJECTION PROCEDURE

The demand projection procedure described in the previous section, will be illustrated by using aluminum as an example in this section. Aluminum is chosen for the illustration because it is one of the important mineral commodities and has a large variety of end-use applications. Also, it was one of the commodities used in our earlier prototype studies for testing the method.

Compilation of Data

Aluminum consumption was disaggregated into 11 end uses. The annual data for each end use are shown in appendix C. At the time the Bureau's statistical projections were made, the available data for the eight metal end uses covered the period 1960-76, and the data for the nonmetal end uses were available only for 1966-76. Recently, the data have been revised. Therefore, there are small differences between these original data and the newer data, published in the "Minerals in the U.S. Economy: Ten-Year Supply-Demand Profiles for Nonfuel Mineral Commodities (1968-77)," which are available for 1968 to 1977.

The annual data of the U.S. macroeconomic explanatory variables used in estimating the ordinary least-square equations of aluminum end-use consumption are shown in appendix D. These data are obtained from various Government sources as indicated in the previous section.

Selection of Independent Variable

The selection of the macroeconomic variable that was used in each estimated equation was done in consultation with the aluminum specialist. The number of explanatory variables chosen varied from as many as 10 for the transportation end use to as few as 5 for the cans and containers end use, as shown in appendix E.

Selection of Estimated Equation Used for Projection

As shown in appendix E, the explanatory variable with the highest R^2 value in each end use is indicated by an asterisk. For example, in the construction end-use category, the results of the R^2 value range from 0.72 to 0.95, and the explanatory variable with the highest R^2 value is U.S. new construction activity. This means that 95 percent of the variation (about the mean) in the consumption of aluminum used in construction has been explained by the linear influence of U.S. new construction activity. Therefore, among the six estimated equations, the following estimated equation with U.S. new construction activity as the explanatory variable was selected for deriving the statistical demand projection for aluminum used in construction:

Construction (data period 1960-76)

$$Y_t = -1,301.0015 + 20.9926X_t \quad (5)$$

$$R^2 = 0.9549$$

where Y_t = aluminum used in construction at time t , thousand short tons,

and X_t = U.S. new construction activity at time t , billion 1973 dollars.

For other end uses, the following estimated equations were selected:

Transportation (data period 1960-76)

$$Y_t = -171.8997 + 7.5565X_t \quad (6)$$

$$R^2 = 0.9272$$

where Y_t = aluminum used in transportation at time t , thousand short tons,

and X_t = FRB industrial production index for motor vehicles at time t , 1967 = 100.

Electrical (data period 1960-76)

$$Y_t = -211.9089 + 7.7377X_t \quad (7)$$

$$R^2 = 0.9562$$

where Y_t = aluminum for electrical use at time t , thousand short tons,

and X_t = FRB industrial production index for major electrical equipment and parts at time t , 1967 = 100.

Cans and containers (data period 1960-76)

$$Y_t = -943.3190 + 13.9370X_t \quad (8)$$

$$R^2 = 0.9473$$

where Y_t = aluminum used in cans and containers at time t , thousand short tons,

and X_t = FRB industrial production index for metal cans at time t , 1967 = 100.

Appliances and equipment (data period 1960-76)

$$Y_t = -145.9117 + 3.3436X_t \quad (9)$$

$$R^2 = 0.9647$$

where Y_t = aluminum used in appliances and equipment at time t, thousand short tons,

and X_t = gross private domestic investment at time t, billion 1973 dollars.

Machinery (data period 1960-76)

$$Y_t = -10.8506 + 3.0758X_t \quad (10)$$

$$R^2 = 0.8986$$

where Y_t = aluminum used in machinery at time t, thousand short tons,

and X_t = FRB industrial production index for nonelectrical machinery at time t, 1967 = 100.

Dissipative (data period 1960-76)

$$Y_t = -131.2343 + 2.6701X_t \quad (11)$$

$$R^2 = 0.7827$$

where Y_t = aluminum for dissipative use at time t, thousand short tons,

and X_t = gross private domestic investment at time t, billion 1973 dollars.

Other metal uses (data period 1960-76)

$$Y_t = 176.3138 - 0.6843X_t \quad (12)$$

$$R^2 = 0.1932$$

where Y_t = aluminum for other metal uses at time t, thousand short tons,

and X_t = FRB industrial production index for chemicals and products at time t, 1967 = 100.

Refractories (data period 1966-76)

$$Y_t = -165.1793 + 2.9170X_t \quad (13)$$

$$R^2 = 0.7288$$

where Y_t = aluminum used in refractories at time t,
thousand short tons,

and X_t = FRB industrial production index for stone,
clay, and glass at time t, 1967 = 100.

Chemicals (data period 1966-76)

$$Y_t = -336.7366 + 5.2409X_t \quad (14)$$

$$R^2 = 0.8248$$

where Y_t = aluminum used in chemicals at time t,
thousand short tons,

and X_t = FRB industrial production index for paper
and products at time t, 1967 = 100.

Abrasives (data period 1966-76)

$$Y_t = -86.8939 + 1.5614X_t \quad (15)$$

$$R^2 = 0.6393$$

where Y_t = aluminum used in abrasives at time t,
thousand short tons,

and X_t = FRB total industrial production index at
time t, 1967 = 100.

Determination of End-Use Projection

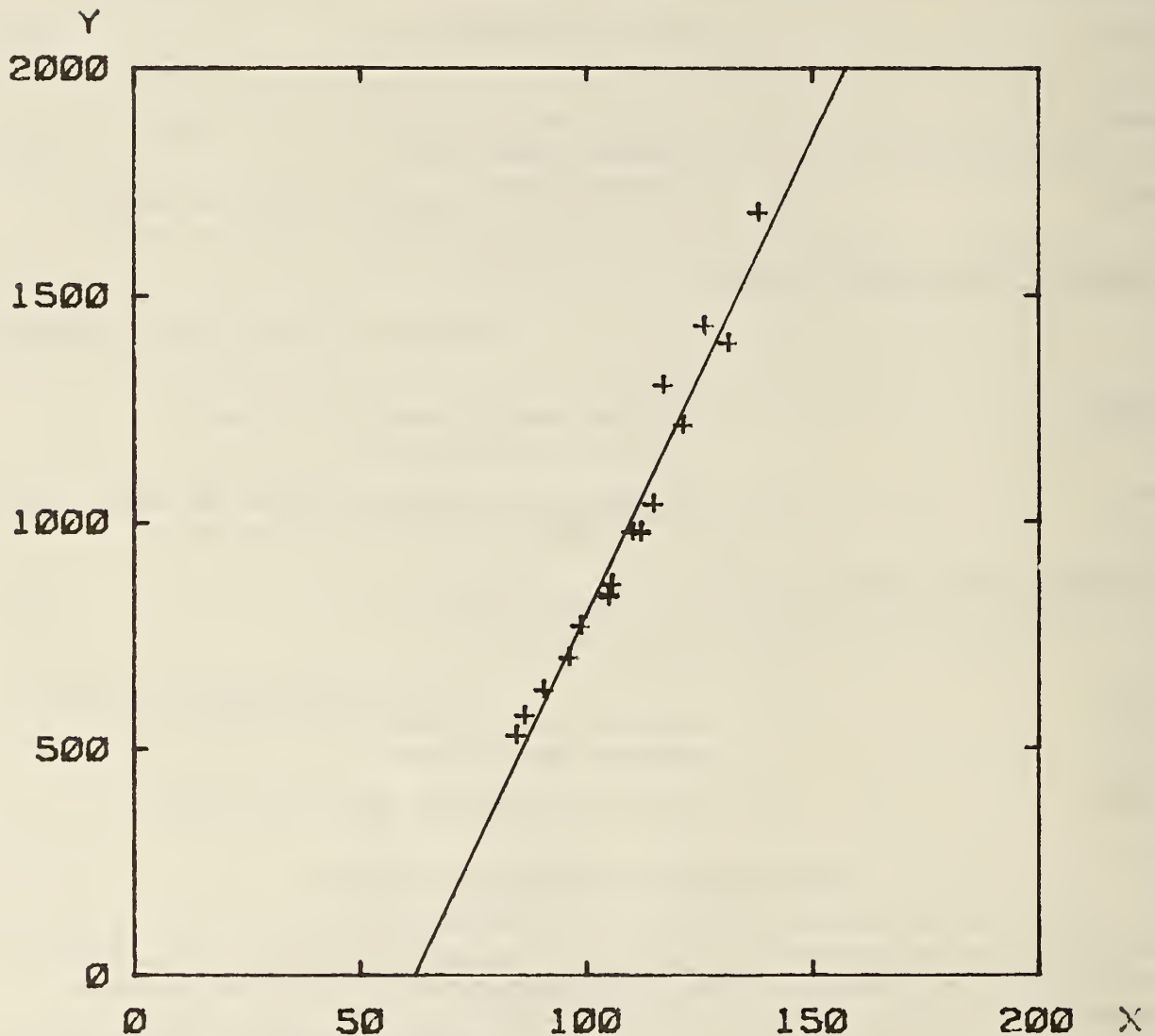
After the appropriate estimated equation has been selected, the end-use projection can be obtained by solving the estimated equation for the dependent variable by substituting the appropriate period's value for the macro-economic explanatory variable in the estimated equation. For example, the projection of aluminum consumption in the construction end use for year 2000 is 5,375,000 short tons, which is calculated from estimated equation 5 under the assumption that the value of U.S. new construction activity in year 2000 will be \$318 billion; that is--

$$\begin{aligned} Y_{2000} &= -1,301.0015 + 20.9926X_{2000} \\ &= -1,301.0015 + (20.9926)(318) \\ &= -1,301.0015 + 6,675.6468 \\ &= 5,374.6 \text{ thousand short tons} \end{aligned}$$

By using the same procedure, the other end-use projections can also be obtained.

Figures 1 through 11 are the computer graphs corresponding to the estimated equations 5 through 15 for the 11 end uses. Sixty-seven similar graphs with smaller R^2 's appear in appendix F.

ALUMINUM DEMAND--CONSTRUCTION
THOUSAND SHORT TONS (1960-1976)



NEW CONSTRUCTION ACTIVITY
BILLION 1973 \$

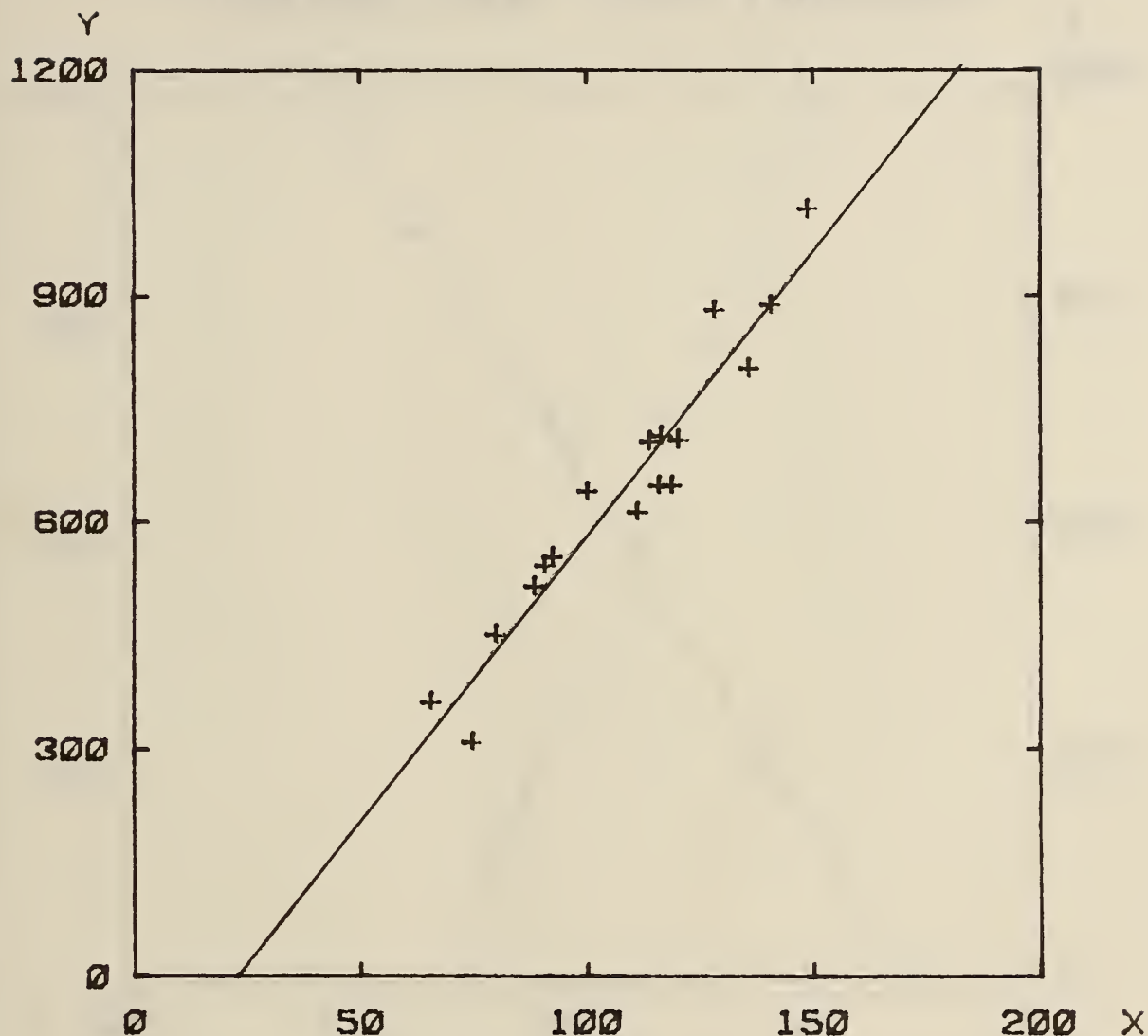
$$Y = -1301.0015 + 20.9926 X$$

$$R\text{-SQUARED} = 0.9549$$

PROJECTIONS	Y	X
1985	2981.5008	204.0000
2000	5374.6639	318.0000

FIGURE 1. - Scatter diagram of aluminum demand for construction use and new construction activity.

ALUMINUM DEMAND--TRANSPORTATION THOUSAND SHORT TONS (1960-1976)



FRBI-MOTOR VEHICLES
1967 = 100

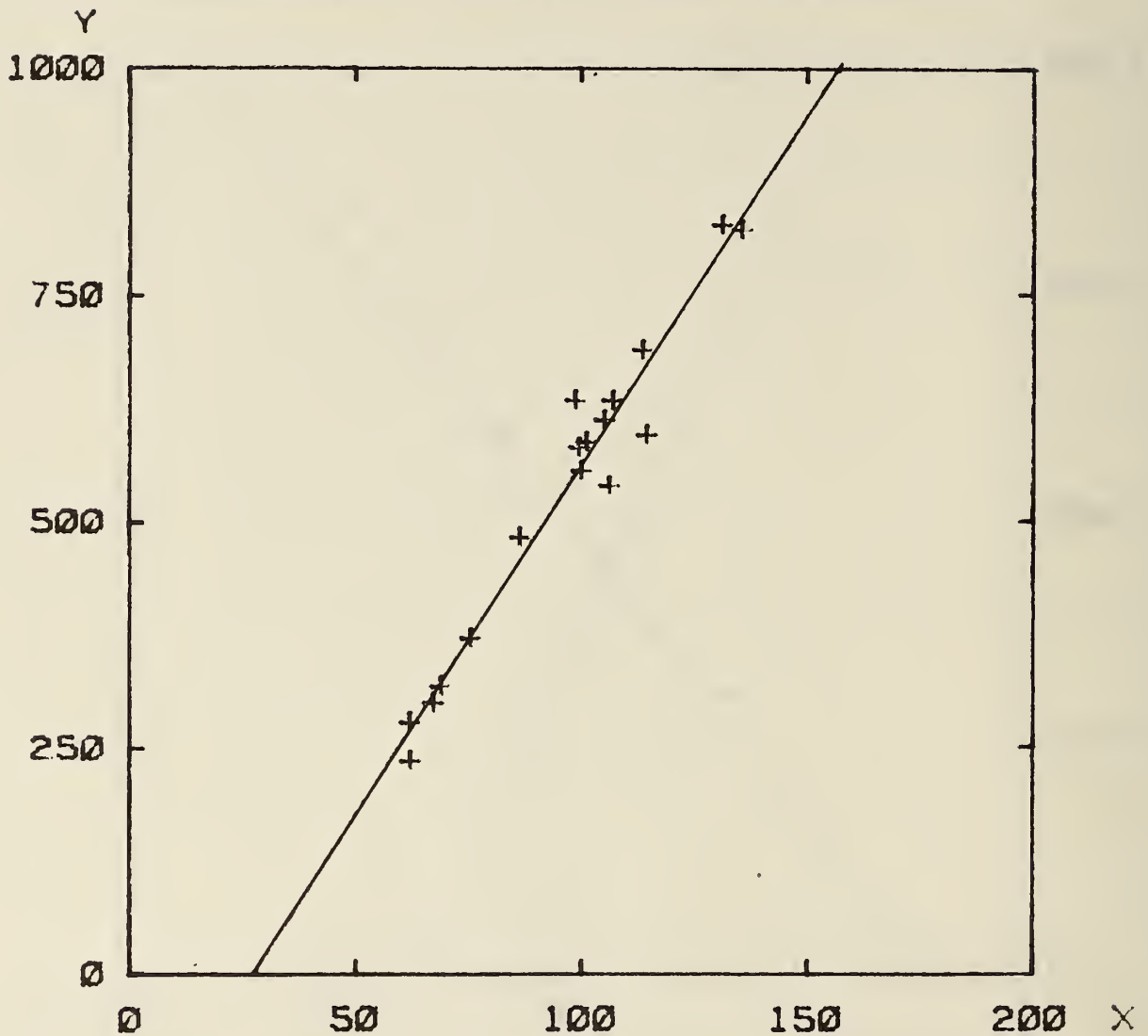
$$Y = -171.8997 + 7.5565 X$$

$$R\text{-SQUARED} = 0.9272$$

PROJECTIONS	Y	X
1985	1201.8743	181.8000
2000	2033.0907	291.8000

FIGURE 2. - Scatter diagram of aluminum demand for transportation use and FRB index for motor vehicles.

ALUMINUM DEMAND--ELECTRICAL
THOUSAND SHORT TONS (1960-1976)



FRBI-MAJOR ELECTRICAL EQUIPMENT & PARTS
1967 = 100

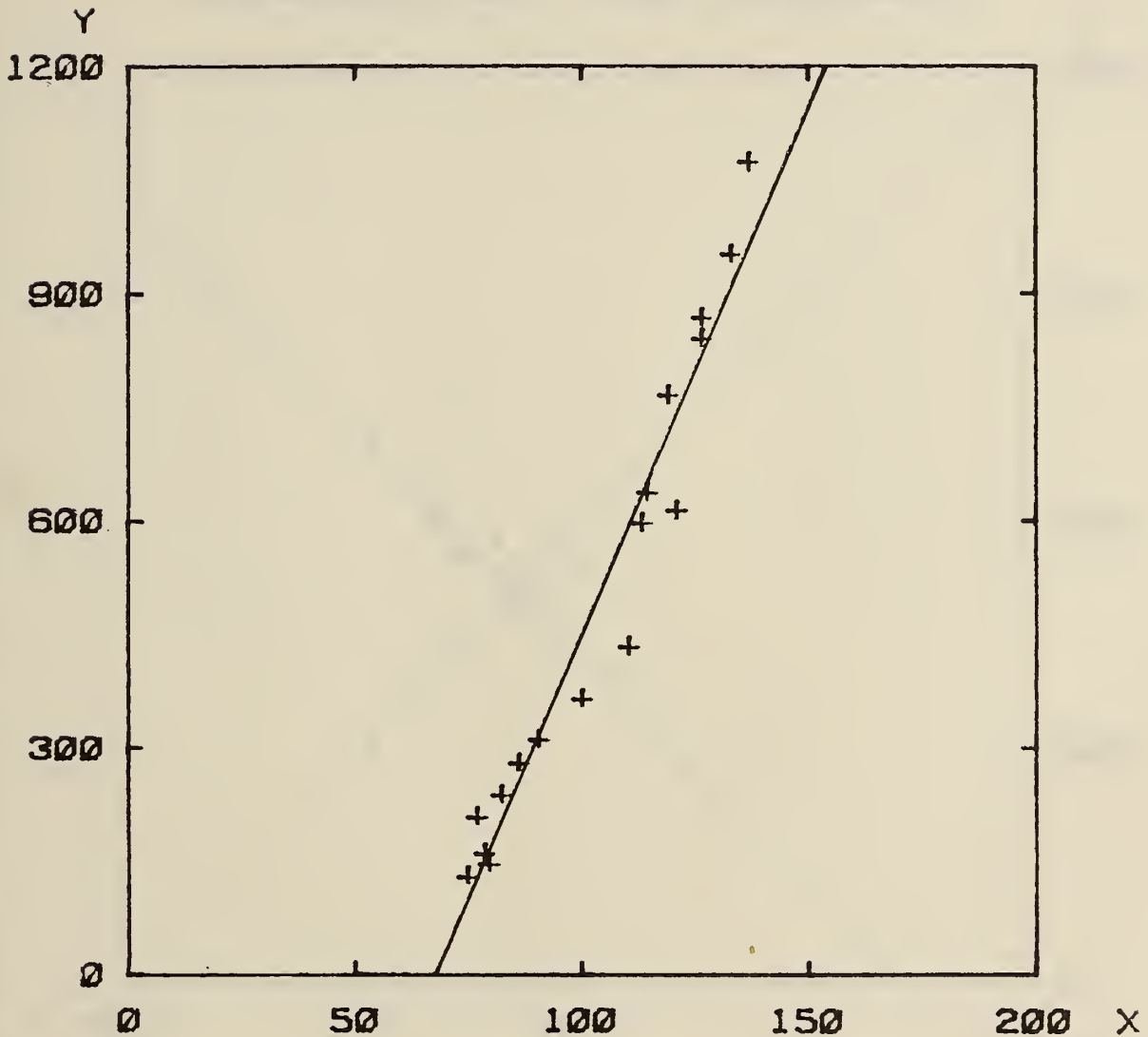
$$Y = -211.8088 + 7.7377 X$$

$$R\text{-SQUARED} = 0.9562$$

PROJECTIONS	Y	X
1985	1129.0498	173.3000
2000	1560.0446	229.0000

FIGURE 3. - Scatter diagram of aluminum demand for electrical use and FRB index for major electrical equipment and parts.

ALUMINUM DEMAND--CANS AND CONTAINERS
THOUSAND SHORT TONS (1960-1976)



FRBI-METAL CANS
1967 = 100

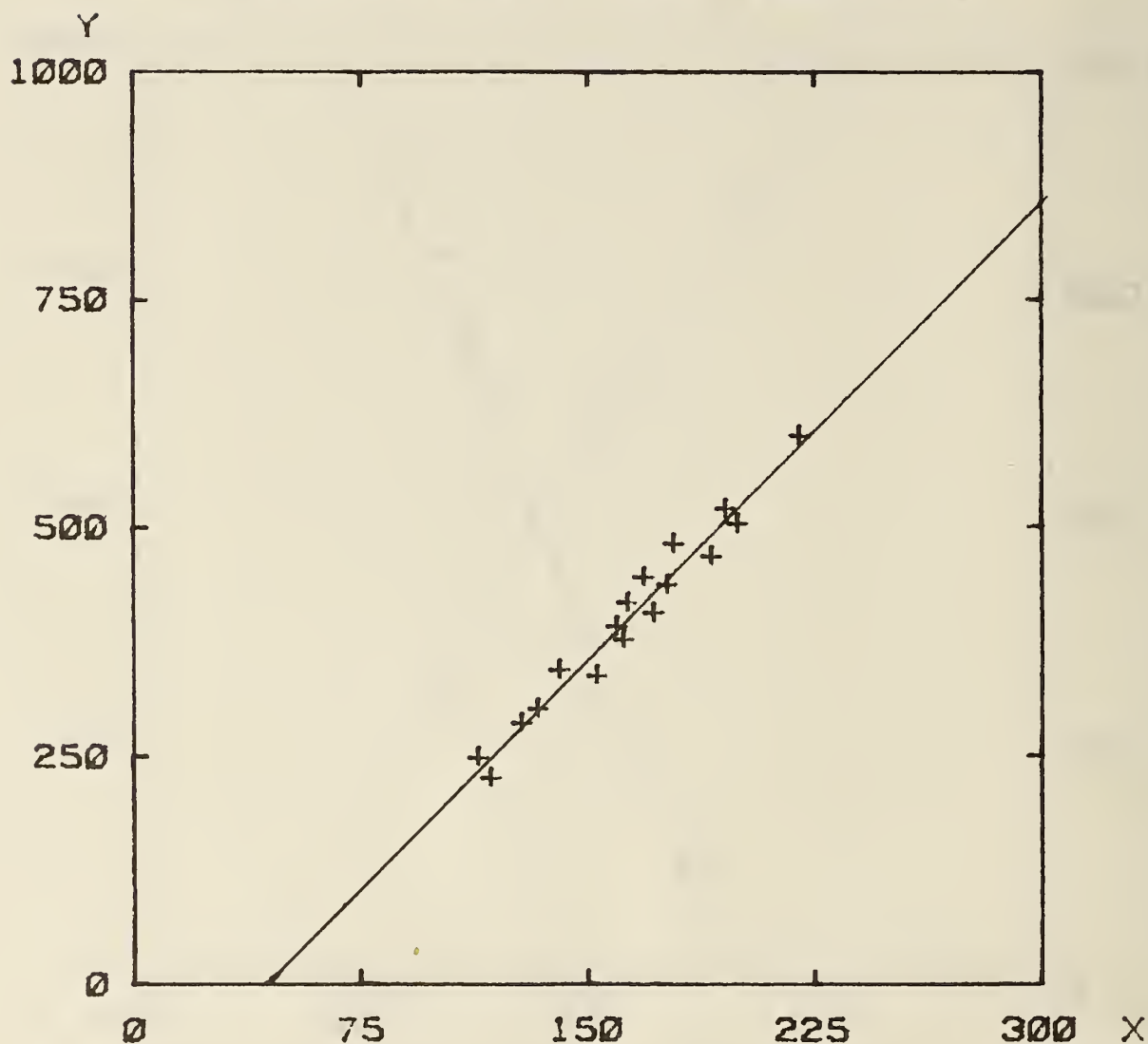
$$Y = -943.3180 + 13.9370 X$$

R-SQUARED = 0.8473

PROJECTIONS	Y	X
1985	1414.8263	169.2000
2000	1841.6460	207.0000

FIGURE 4. - Scatter diagram of aluminum demand for cans and containers use and FRB index for metal cans.

ALUMINUM DEMAND--APPLIANCES AND EQUIPMENT THOUSAND SHORT TONS (1980-1976)



GROSS PRIVATE DOMESTIC INVESTMENT
BILLION 1973 \$

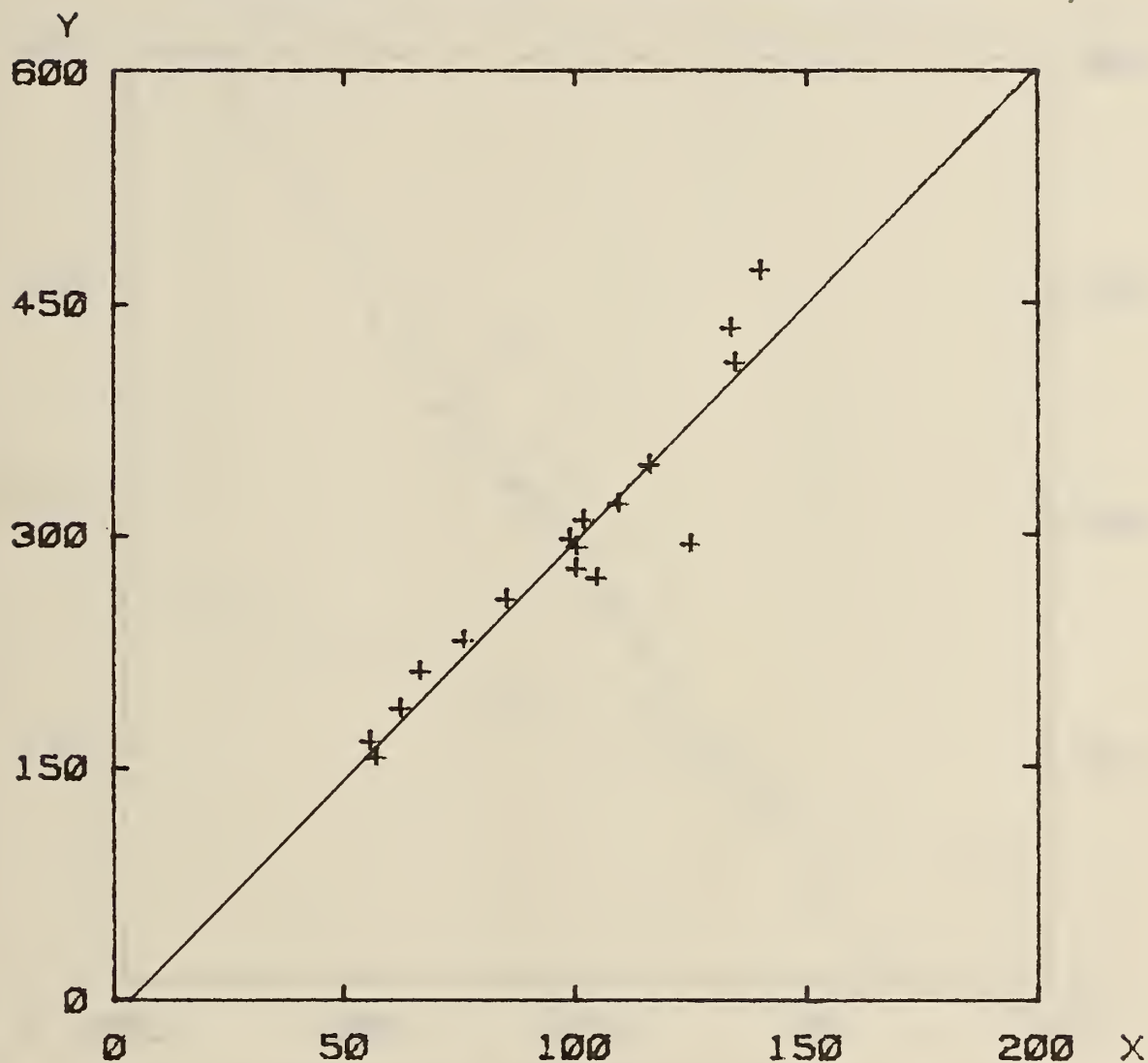
$$Y = -145.9117 + 3.3436 X$$

$$R\text{-SQUARED} = 0.9647$$

PROJECTIONS	Y	X
1985	903.9932	314.0000
2000	1592.7843	520.0000

FIGURE 5. - Scatter diagram of aluminum demand for appliances and equipment use and gross private domestic investment.

ALUMINUM DEMAND--MACHINERY
THOUSAND SHORT TONS (1960-1976)



FRBI-NONELECTRICAL MACHINERY
1967 = 100

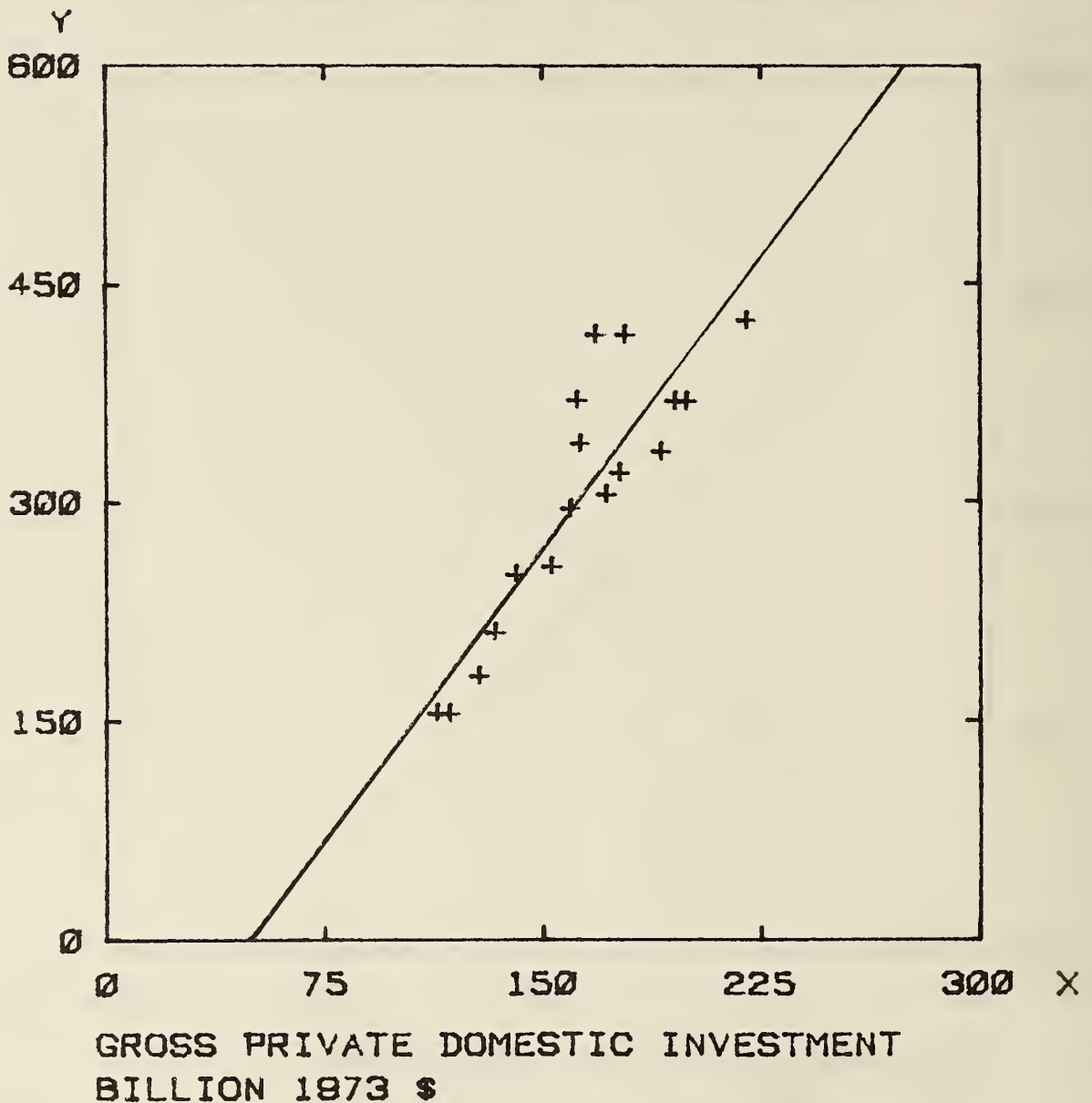
$$Y = -10.8506 + 3.0758 X$$

$$R\text{-SQUARED} = 0.8986$$

PROJECTIONS	Y	X
1985	606.7728	200.8000
2000	979.5614	322.0000

FIGURE 6. - Scatter diagram of aluminum demand for machinery use and FRB index for nonelectrical machinery.

ALUMINUM DEMAND--DISSIPATIVE
THOUSAND SHORT TONS (1960-1976)



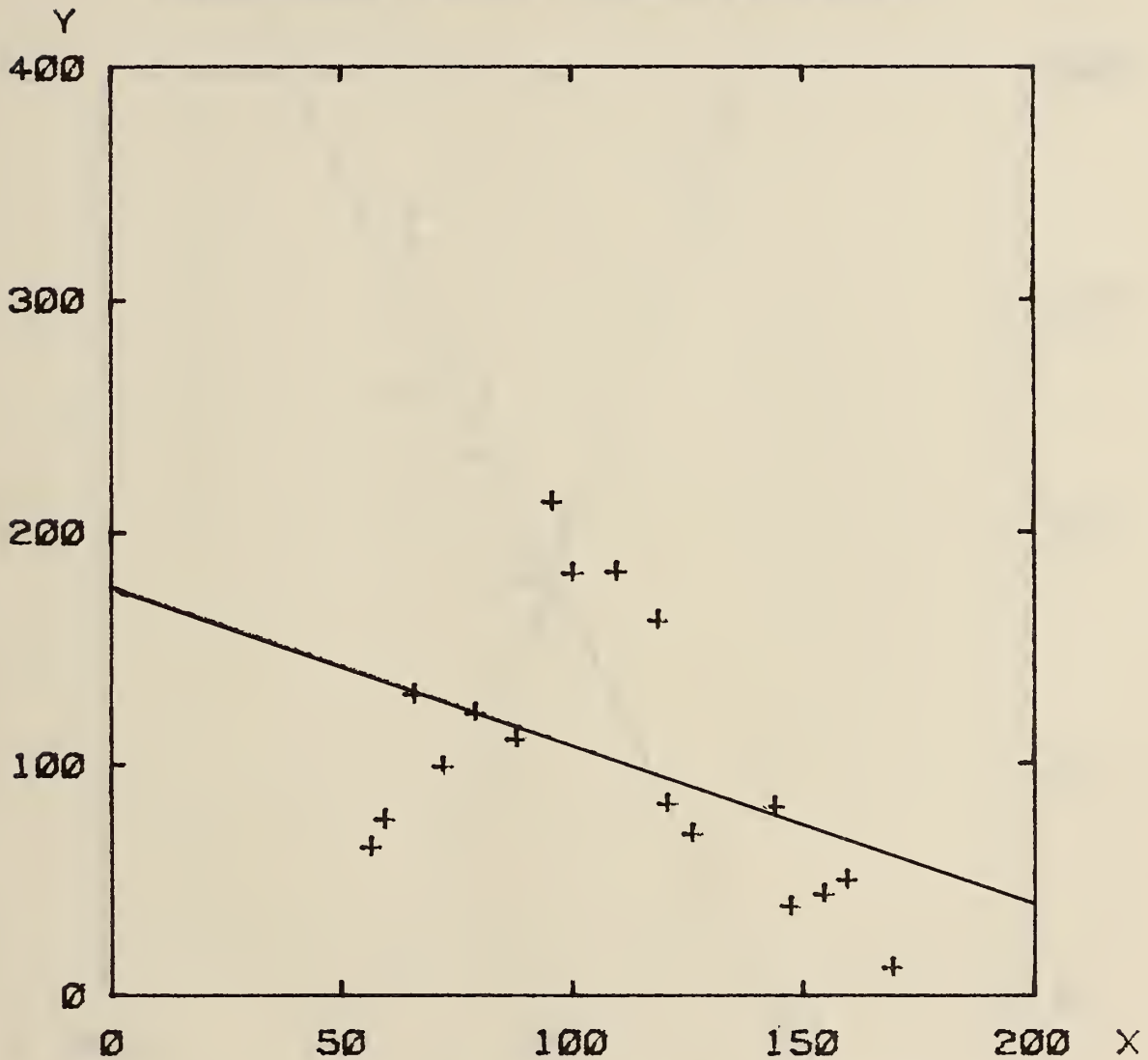
$$Y = -131.2343 + 2.6701 X$$

$$R\text{-SQUARED} = 0.7827$$

PROJECTIONS	Y	X
1985	707.1915	314.0000
2000	1257.2416	520.0000

FIGURE 7. - Scatter diagram of aluminum demand for dissipative use and gross private domestic investment.

ALUMINUM DEMAND--OTHER METAL
THOUSAND SHORT TONS (1960-1976)



FRBI-CHEMICALS AND PRODUCTS
1967 = 100

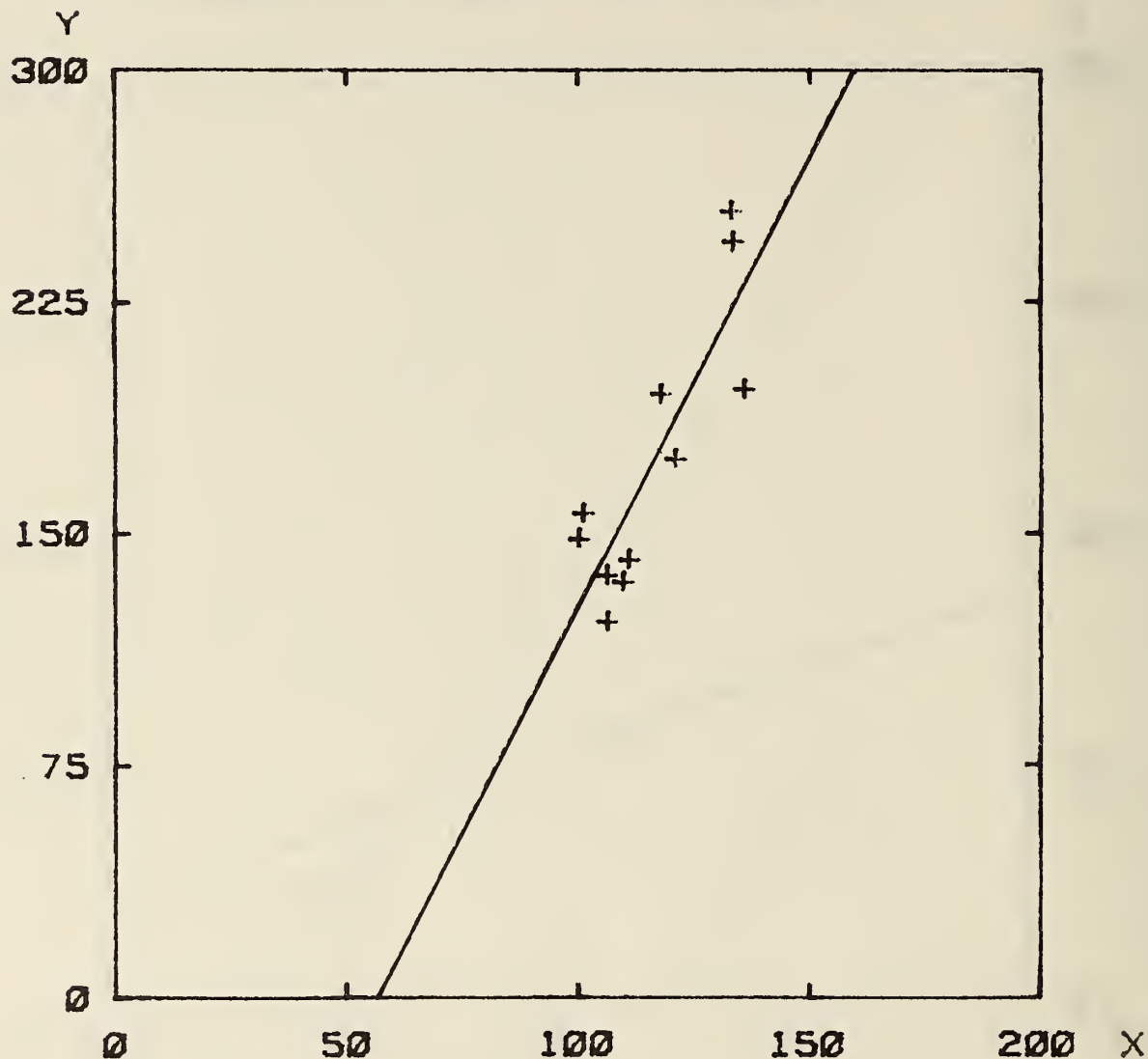
$$Y = 176.3138 - 0.6843 X$$

$$R\text{-SQUARED} = 0.1932$$

PROJECTIONS		Y	X
1985	-	23.8447	292.5000
2000	-	263.0086	642.0000

FIGURE 8. - Scatter diagram of aluminum demand for other metal use and FRB index for chemicals and products.

ALUMINUM DEMAND--REFRACTORIES
THOUSAND SHORT TONS (1960-1976)



FRBI-STONE, CLAY, AND GLASS
1967 = 100

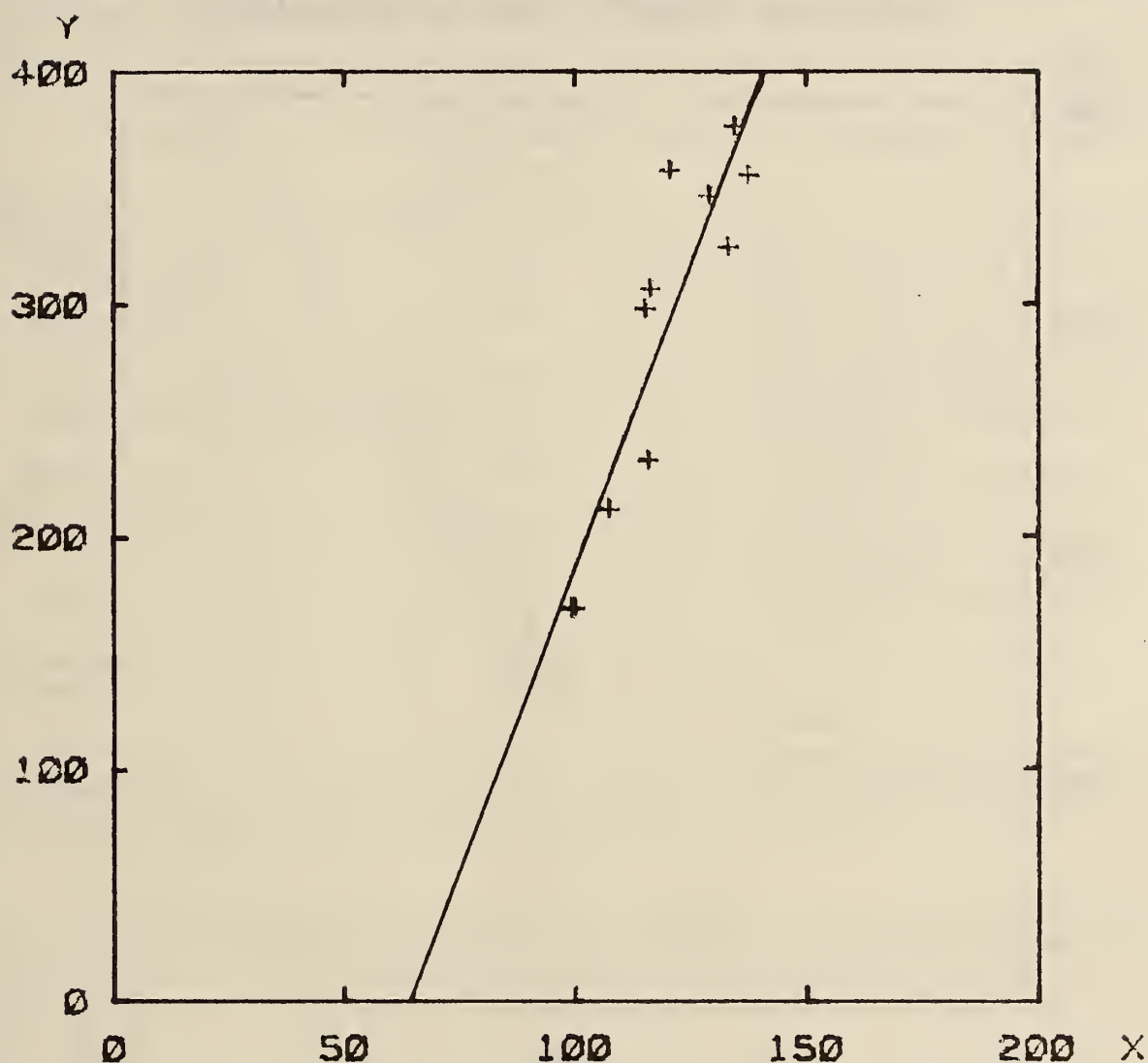
$$Y = -165.1793 + 2.9170 X$$

$$R\text{-SQUARED} = 0.7288$$

PROJECTIONS	Y	X
1985	342.0891	173.9000
2000	616.5954	268.0000

FIGURE 9. - Scatter diagram of aluminum demand for refractories use and FRB index for stone, clay, and glass.

ALUMINUM DEMAND--CHEMICALS
THOUSAND SHORT TONS (1960-1976)



FRBI-PAPER AND PRODUCTS
1967 = 100

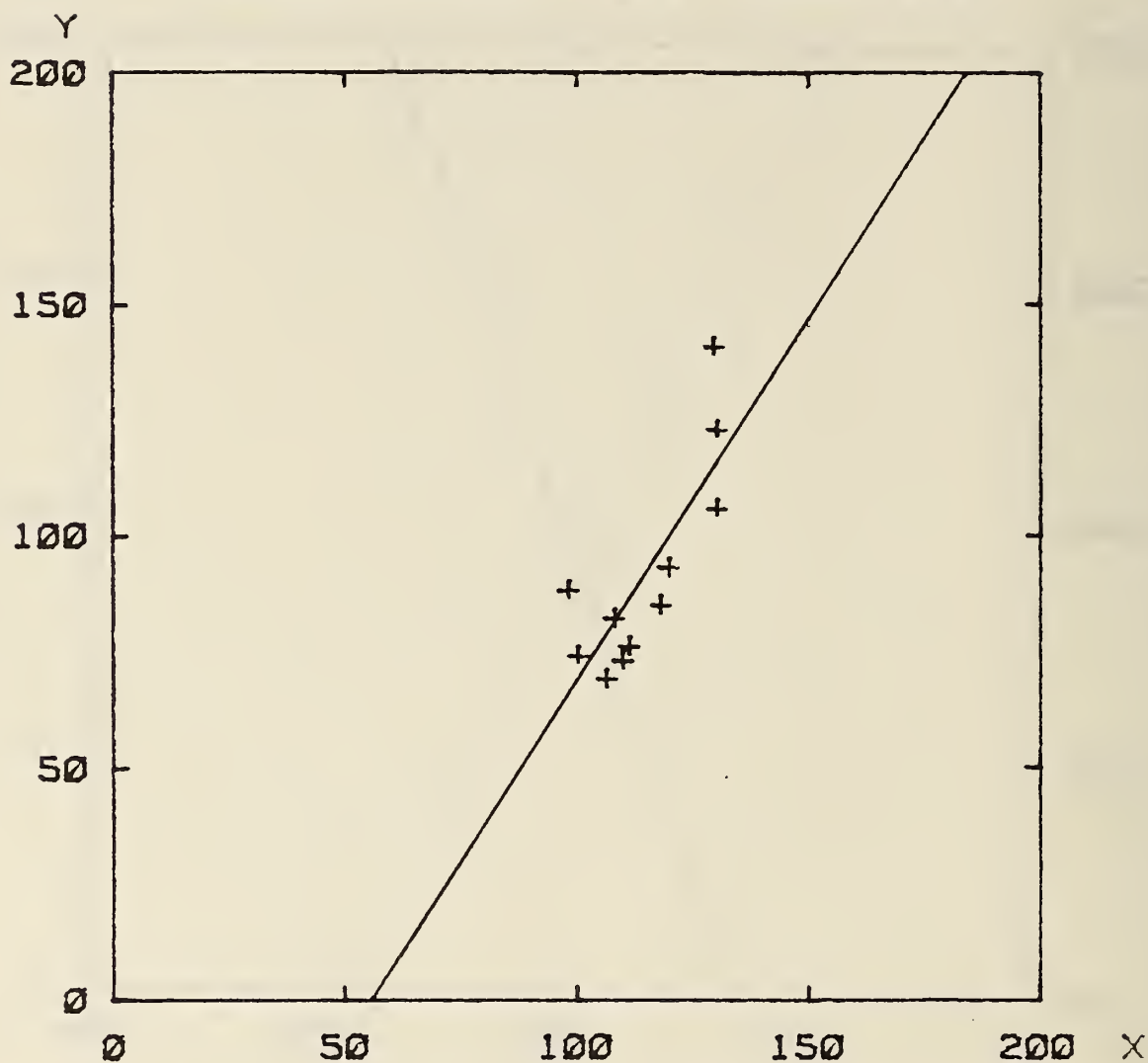
$$Y = - 336.7366 + 5.2409 X$$

$$R\text{-SQUARED} = 0.8248$$

PROJECTIONS	Y	X
1985	725.5947	202.7000
2000	1240.7756	301.0000

FIGURE 10. - Scatter diagram of aluminum demand for chemicals use and FRB index for paper and products.

ALUMINUM DEMAND--ABRASIVES
THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION
1967 = 100

$$Y = -86.8939 + 1.5614 X$$

$$R\text{-SQUARED} = 0.6393$$

PROJECTIONS	Y	X
1985	203.9994	186.3000
2000	397.1476	310.0000

FIGURE 11. - Scatter diagram of aluminum demand for abrasives use and FRB index for total production.

The following tabulation summarizes the statistical projections and associated R^2 values and explanatory variables:

End Use	Statistical projections, year 2000 (thousand short tons)	R^2 value	Explanatory variable
Metal:			
Construction.....	5,375	0.9549	New construction activity.
Transportation.....	2,033	.9272	Motor vehicles index. ¹
Electrical.....	1,560	.9562	Major electrical equipment and parts index. ¹
Cans and containers.....	1,942	.9473	Cans and containers index. ¹
Appliances and equipment	1,593	.9647	Gross private domestic investment.
Machinery.....	980	.8986	Nonelectrical machinery index. ¹
Dissipative.....	1,257	.7827	Gross private domestic investment.
Other.....	0	.1932	Chemicals and products index. ¹
Nonmetal:			
Refractories.....	617	.7288	Stone, clay, and glass index. ¹
Chemicals.....	1,241	.8248	Paper and products index. ¹
Abrasives.....	397	.6393	Total production index. ¹

¹All indexes are Federal Reserve Board industrial production indexes (1967 = 100).

In order to compare the historical consumption of each end use with the statistical projections for 1985 and year 2000, the statistical demand projection system also provides the following computer graphs, figures 12 through 22.

ALUMINUM DEMAND--CONSTRUCTION
THOUSAND SHORT TONS (1960-1976)

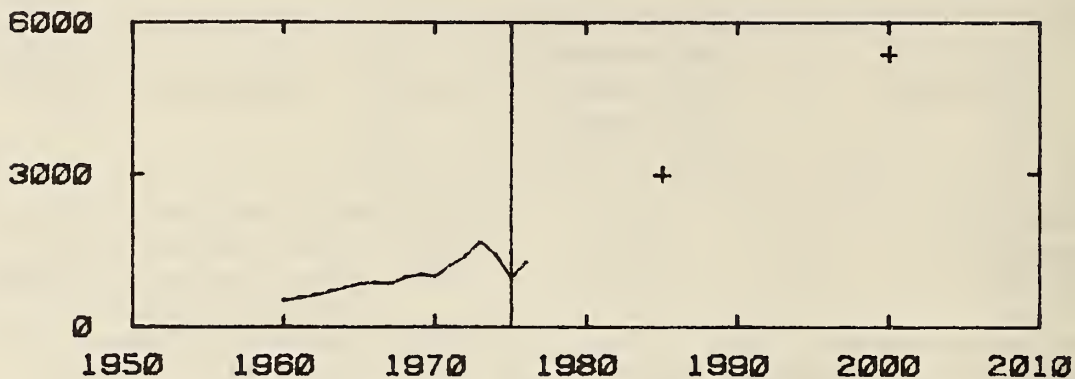


FIGURE 12. - Aluminum demand for construction use.

ALUMINUM DEMAND--TRANSPORTATION
THOUSAND SHORT TONS (1960-1976)

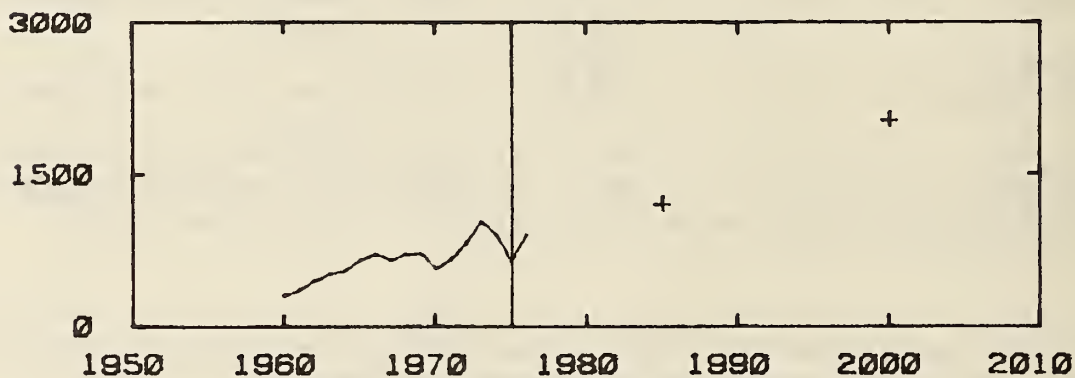


FIGURE 13. - Aluminum demand for transportation use.

ALUMINUM DEMAND--ELECTRICAL
THOUSAND SHORT TONS (1960-1976)

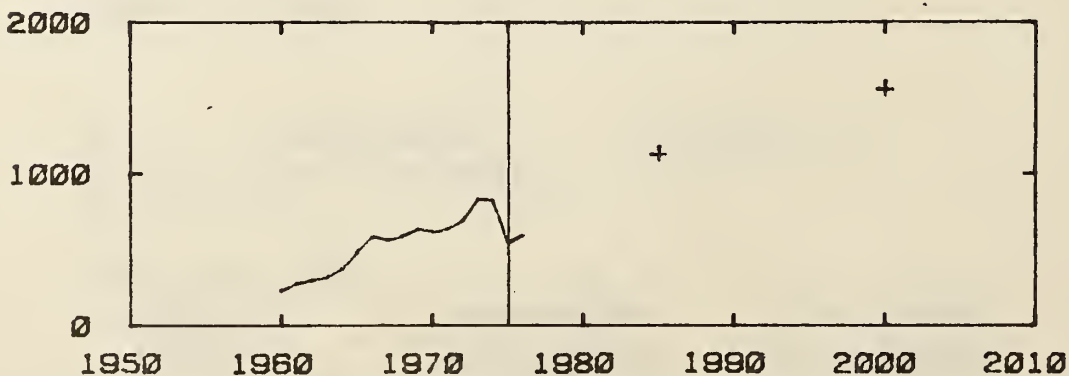


FIGURE 14. - Aluminum demand for electrical use.

ALUMINUM DEMAND--CANS AND CONTAINERS
THOUSAND SHORT TONS (1960-1976)

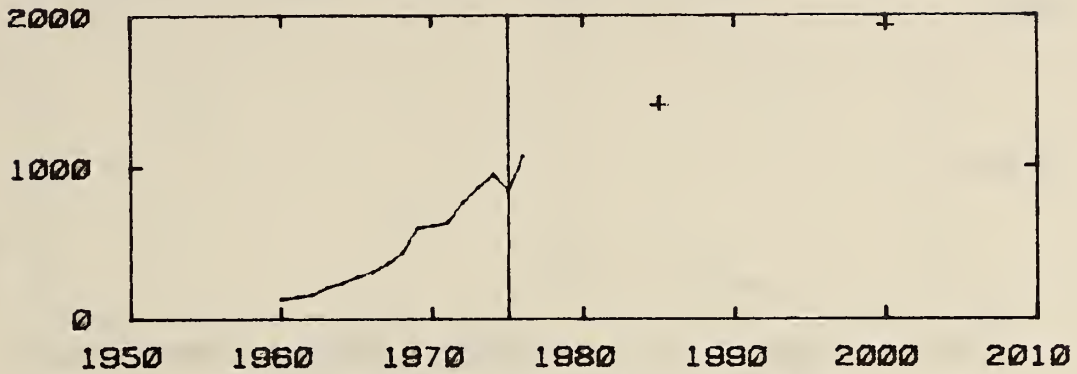


FIGURE 15. - Aluminum demand for cans and containers use.

ALUMINUM DEMAND--APPLIANCES AND EQUIPMENT
THOUSAND SHORT TONS (1960-1976)

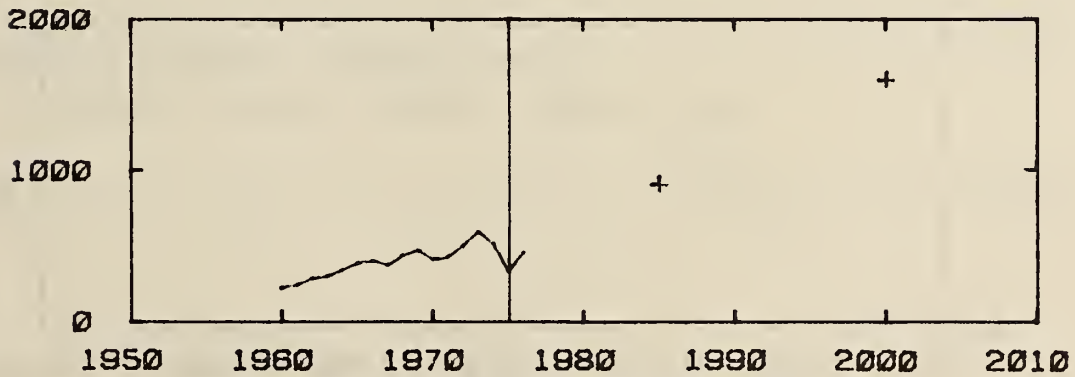


FIGURE 16. - Aluminum demand for appliances and equipment use.

ALUMINUM DEMAND--MACHINERY
THOUSAND SHORT TONS (1960-1976)

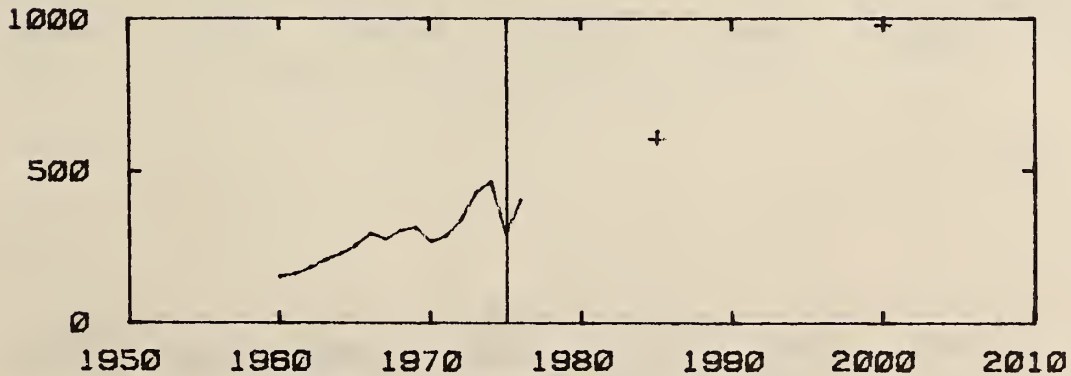


FIGURE 17. - Aluminum demand for machinery use.

ALUMINUM DEMAND--DISSIPATIVE
THOUSAND SHORT TONS (1960-1976)

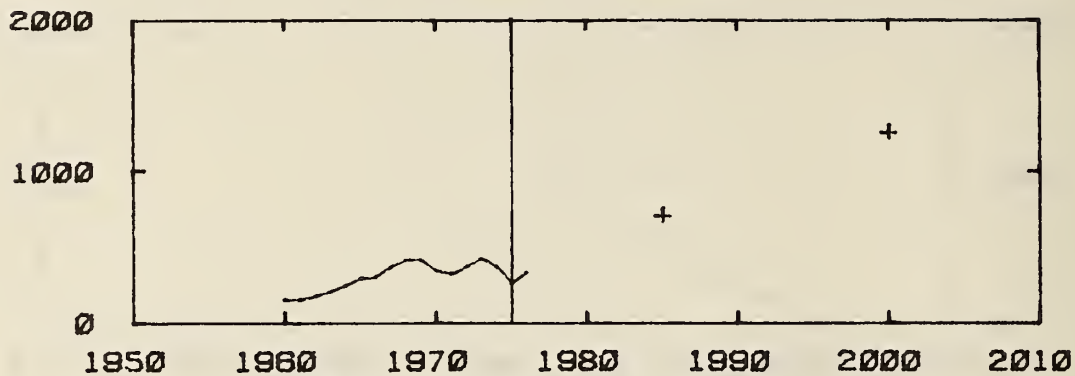


FIGURE 18. - Aluminum demand for dissipative use.

ALUMINUM DEMAND--OTHER METAL
THOUSAND SHORT TONS (1960-1976)

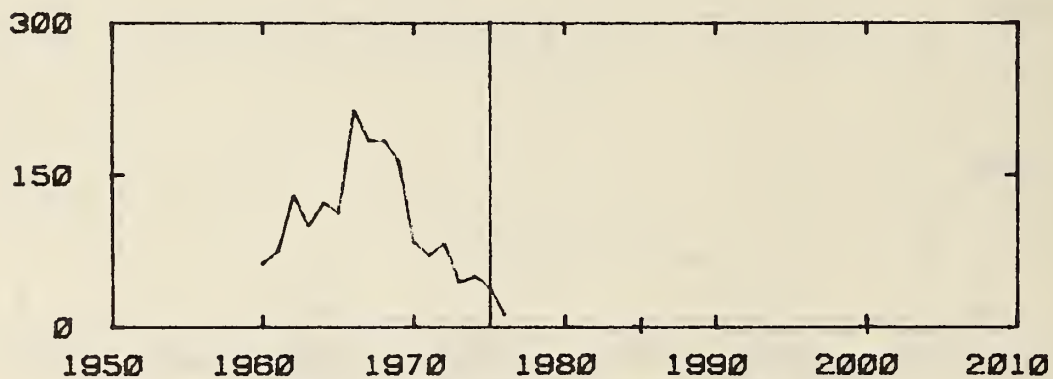


FIGURE 19. - Aluminum demand for other metal use.

ALUMINUM DEMAND--REFRACTORIES
THOUSAND SHORT TONS (1960-1976)

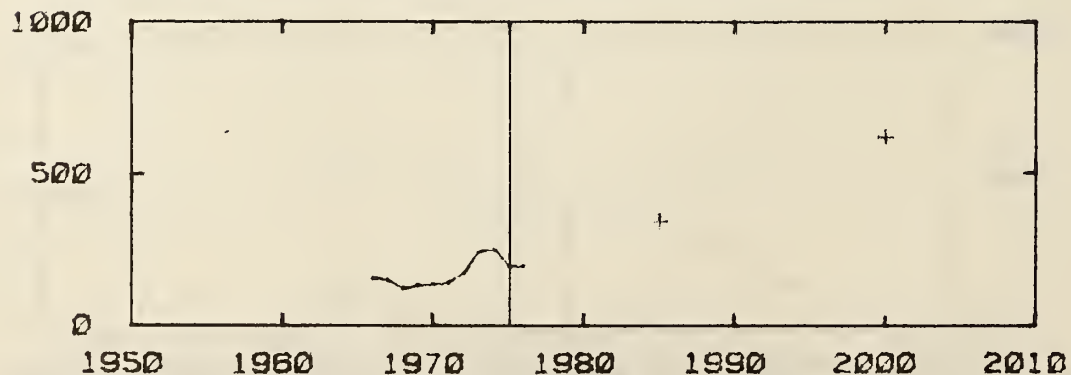


FIGURE 20. - Aluminum demand for refractories use.

ALUMINUM DEMAND--CHEMICALS
THOUSAND SHORT TONS (1960-1976)

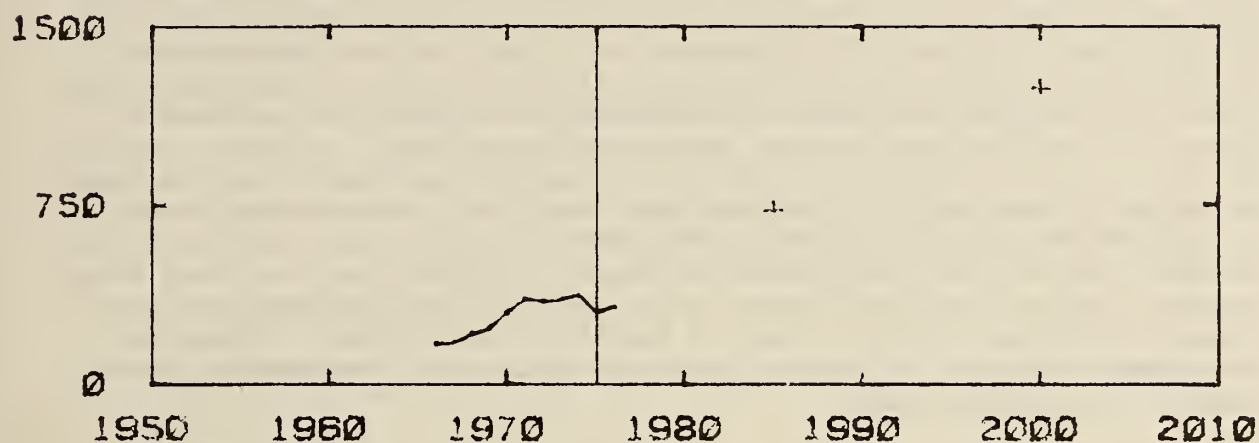


FIGURE 21. - Aluminum demand for chemicals use.

ALUMINUM DEMAND--ABRASIVES
THOUSAND SHORT TONS (1960-1976)

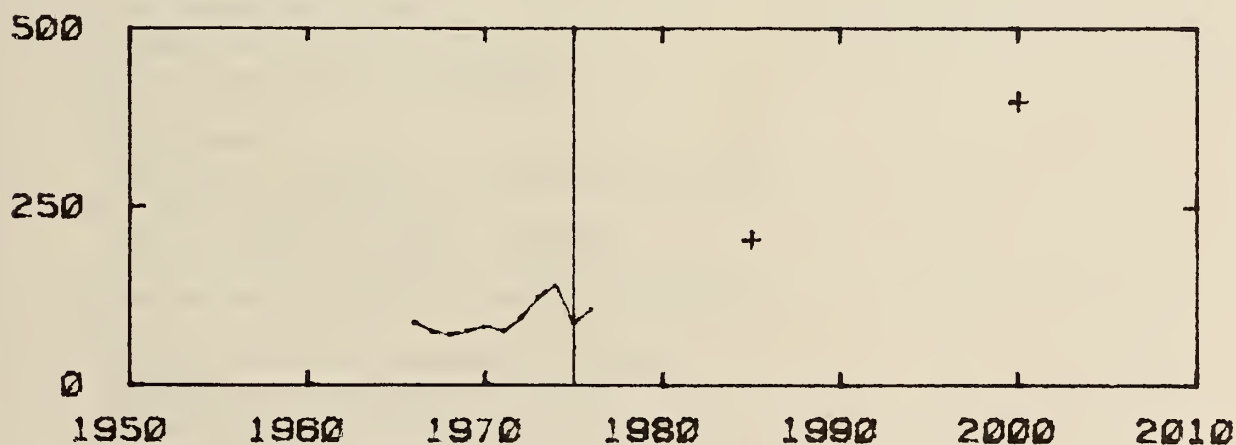


FIGURE 22. - Aluminum demand for abrasives use.

CONCLUSION

The statistical U.S. mineral end-use demand projection system described in this Information Circular is only one component of the Bureau of Mines forecasting system. The results of statistical analyses obtained from the system provide very useful information as to which related macroeconomic variable best explains each historical pattern of end-use consumption, and, therefore, which macroeconomic variable should be used as an explanatory factor for deriving each end-use statistical projection. To derive the projection, a set of consistent estimated values for explanatory macroeconomic variables is obtained from the Data Resources, Inc., macroeconomic model. In the context of using regression analysis for projection purposes, it is assumed that the past empirical relationship will continue, but since there are many reasons this relationship might not hold true in the future, contingency analyses to derive a set of forecasts are necessary. Thus, the contingency analysis is also an important component of the Bureau of Mines demand forecasting system.

Under the current system, the end-use statistical projections and contingency forecasts for each nonfuel mineral commodity in the year 2000 are updated and revised as new data or improved techniques become available.

APPENDIX A.--MACROECONOMIC VARIABLES USED IN BUREAU OF MINES
STATISTICAL DEMAND PROJECTION SYSTEM

1. U.S. gross national product
2. U.S. population
3. U.S. gross private domestic investment
4. U.S. new construction activity
5. The following Federal Reserve Board indexes of industrial production:

- Total production
- Textile mill products
- Paper and products
- Chemicals and products
 - Basic chemicals
 - Synthetic materials
 - Paints
- Petroleum products
- Rubber and plastics products
 - Tires
 - Rubber excluding tires
 - Plastics products, n.e.c.
- Stone, clay, and glass
- Iron and steel
 - Basic steel and mill products
- Fabricated metal products
 - Metal cans
 - Hardware, plumbing, and structural metal
- Nonelectrical machinery
 - Construction and allied equipment
 - Metalworking machinery
 - Special and general industrial equipment
- Electrical machinery
 - Major electrical equipment and parts
 - Household appliances
 - Communication equipment
- Transportation equipment
 - Motor vehicles and parts
 - Automobiles
 - Trucks, buses, and trailers
 - Aircraft and parts
 - Ships and boats
- Food and products
- Ordnance

APPENDIX B.--LEAST-SQUARES METHOD, BEST UNBIASED ESTIMATOR,
AND MAXIMUM LIKELIHOOD ESTIMATOR

The purpose of appendix B is to show how the least-squares formulas are obtained and how they are related to the "best unbiased estimator" and the "maximum likelihood" estimator.

The simplest form of regression analysis assumes that the data points can be represented by the following linear equation:

$$Y_t = a + b X_t + e_t \quad (\text{for } t=1,2,\dots,n), \quad (\text{B-1})$$

where

Y_t = the dependent variable at time t ,

X_t = the explanatory (or independent) variable at time t ,

and

e_t = the residual (or error term) at time t .

In the formulation of equation B-1, two parameters, a and b , have to be estimated from the actual observations. These parameters can be estimated by many methods dependent upon the underlying assumptions and criteria. Three separate criteria often used are "least-squares," "best linear unbiased," and "maximum likelihood."

The "Least-Squares" Criterion

By the "least-squares" criterion, equation B-1 is fitted to the data points such that the sum of the squares of the residuals is minimized. From equation B-1, we have the sum of the squares of the residuals as follows:

$$\sum_{t=1}^n e_t^2 = \sum_{t=1}^n [Y_t - a - bX_t]^2.$$

Therefore, the minimization of the sum of the squared residuals is equivalent to the minimization of the following term:

$$\sum_{t=1}^n [Y_t - a - bX_t]^2. \quad (\text{B-2})$$

Using the theory of calculus, the procedures for finding the estimates of a and b such that the value of equation B-2 is minimized are

1. Taking the partial derivative of equation B-2 with respect to the parameters a and b respectively and setting them equal to zero:

$$\frac{\partial(\text{B-2})}{\partial a} = 0 \quad (\text{B-3})$$

and

$$\frac{\partial(\text{B-2})}{\partial b} = 0. \quad (\text{B-4})$$

2. Obtaining the estimate of the parameters a and b by solving equations B-3 and B-4 simultaneously.

By procedure 1, we have

$$-2 \sum_{t=1}^n [Y_t - a - bX_t] = 0 \quad (B-5)$$

and

$$-2 \sum_{t=1}^n [Y_t - a - bX_t] X_t = 0. \quad (B-6)$$

Solving for a from equation B-5, we get

$$a = (1/n) \sum_{t=1}^n [Y_t - bX_t]. \quad (B-7)$$

Substituting equation B-7 into B-6 and solving for b, we obtain the estimate of b as follows:

$$\begin{aligned} \hat{b} &= \left[\sum_{t=1}^n X_t Y_t - (1/n) \sum_{t=1}^n X_t \sum_{t=1}^n Y_t \right] / \left[\sum_{t=1}^n X_t^2 - (1/n) \sum_{t=1}^n X_t \sum_{t=1}^n X_t \right] \\ &= \left[\sum_{t=1}^n X_t Y_t - n \bar{X} \bar{Y} \right] / \left[\sum_{t=1}^n X_t^2 - n \bar{X}^2 \right] \end{aligned} \quad (B-8)$$

$$\text{where } \bar{X} = (1/n) \sum_{t=1}^n X_t, \text{ and } \bar{Y} = (1/n) \sum_{t=1}^n Y_t. \quad (B-9)$$

Equation B-7 can be written

$$a = \bar{Y} - b\bar{X}. \quad (B-10)$$

After the estimate of b is obtained from formula B-8, the estimate of the parameter a can be obtained by substituting $\hat{b}$ for b in equation B-10:

$$\hat{a} = \bar{Y} - \hat{b}\bar{X}. \quad (B-11)$$

The "Best Linear Unbiased" Criterion

To apply the "best linear unbiased" criterion, it is necessary to make the following assumptions about the residuals in equation B-1 because "best linear unbiased" is a statistical concept:

$$E[e_t] = 0, \text{ for } t=1,2,3,\dots,n \quad (\text{B-12})$$

and

$$E[e_t e_j] = \begin{cases} 0 & \text{for } t \neq j; t,j=1,2,3,\dots,n \\ S^2 & \text{for } t=j; t,j=1,2,3,\dots,n. \end{cases} \quad (\text{B-13})$$

Condition B-12 assumes the expected value of the residuals is zero for each t , and condition B-13 assumes that all the residuals have the same variance S^2 and zero covariance between different residuals. By the "linear" criterion, the estimate of b is a linear function of the observations on Y , and it can be expressed as follows:

$$b = \sum_{t=1}^n w_t Y_t. \quad (\text{B-14})$$

Equation B-14 is a class of linear functions on Y with weights w_t . According to the "best linear unbiased" criterion, these weights are chosen to satisfy the following conditions:

1. The "unbiased" condition--The expected value of the estimator should be equal to the true parameter to be estimated; that is,

$$E[\hat{b}] = b. \quad (\text{B-15})$$

2. The "best" condition--The variance of the estimator should be minimized; that is,

$$\text{Minimize } V(b). \quad (\text{B-16})$$

Combining equations B-1 and B-14, the linear estimator of b can be rewritten

$$\begin{aligned} \hat{b} &= \sum_{t=1}^n w_t Y_t \\ &= \sum_{t=1}^n w_t (a + bX_t + e_t) \\ &= a \sum_{t=1}^n w_t + b \sum_{t=1}^n w_t X_t + \sum_{t=1}^n w_t e_t. \end{aligned} \quad (\text{B-17})$$

By taking the expected value of equation B-17, we have

$$\begin{aligned} E[\hat{b}] &= E\left[a \sum_{t=1}^n w_t + b \sum_{t=1}^n w_t X_t + \sum_{t=1}^n w_t e_t\right] \\ &= a \sum_{t=1}^n w_t + b \sum_{t=1}^n w_t X_t + \sum_{t=1}^n w_t E[e_t]. \end{aligned} \quad (\text{B-18})$$

Assumption B-12 makes the last term of B-18 equal to zero. Thus, the expected value of the estimator is

$$E[b] = a \sum_{t=1}^n w_t + b \sum_{t=1}^n w_t X_t. \quad (B-19)$$

By comparing equation B-19 with the "unbiased" condition (B-15), it is shown that b is a linear unbiased estimator of b if the following conditions are satisfied:

$$\sum_{t=1}^n w_t = 0 \quad (B-20)$$

and

$$\sum_{t=1}^n w_t X_t = 1. \quad (B-21)$$

Using assumption B-13 and equation B-17, the variance of b is

$$\begin{aligned} V(b) &= V\left(\sum_{t=1}^n w_t e_t\right) \\ &= S^2 \sum_{t=1}^n w_t^2. \end{aligned} \quad (B-22)$$

Since S^2 is a given constant, the value of $V(b)$ will be minimized if the weights are chosen in such a way that the sum of the squares of the weights is minimized. Thus, the construction of a "best linear unbiased" estimator of the parameter b is simply to find a set of weights such that conditions

B-20 and B-21 are satisfied and $\sum_{t=1}^n w_t^2$ is minimized.

This constrained minimization problem can be solved by minimizing the following function:

$$F = \sum_{t=1}^n w_t^2 - 2K \sum_{t=1}^n w_t - 2H\left(\sum_{t=1}^n w_t X_t - 1\right), \quad (B-23)$$

where K and H are Lagrangian multipliers.

Differentiating F with respect to w_t , K , and H and setting them equal to zero gives the equations

$$w_t = K + HX_t \quad \text{for } t=1,2,3,\dots,n, \quad (\text{B-24})$$

$$\sum_{t=1}^n w_t = 0, \quad (\text{B-25})$$

and
$$\sum_{t=1}^n w_t X_t = 1. \quad (\text{B-26})$$

Summing equation B-24 over t yields

$$\begin{aligned} \sum_{t=1}^n w_t &= \sum_{t=1}^n (K + HX_t) \\ &= nK + H \sum_{t=1}^n X_t. \end{aligned} \quad (\text{B-27})$$

Substituting condition B-25 into the left side of B-27, we have

$$\begin{aligned} nK + H \sum_{t=1}^n X_t &= 0 \\ nK &= -H \sum_{t=1}^n X_t \\ K &= -H (1/n) \sum_{t=1}^n X_t \\ K &= -H \bar{X}. \end{aligned} \quad (\text{B-28})$$

Combining equations B-28 and B-24 yields

$$\begin{aligned} w_t &= -H \bar{X} + H X_t \\ &= H(X_t - \bar{X}). \end{aligned} \quad (\text{B-29})$$

By multiplying both sides of equation B-29 by X_t and summing over all t , we get

$$\sum_{t=1}^n w_t X_t = \sum_{t=1}^n [H(X_t - \bar{X})X_t]. \quad (\text{B-30})$$

By condition B-26, the left side of equation B-30 is equal to 1:

$$\begin{aligned}
 1 &= \sum_{t=1}^n [H(X_t - \bar{X})X_t] \\
 1 &= H \sum_{t=1}^n [X_t^2 - \bar{X} X_t] \\
 1 &= H \left[\sum_{t=1}^n X_t^2 - \bar{X} \sum_{t=1}^n X_t \right] \\
 H &= 1 / \left[\sum_{t=1}^n X_t^2 - n \bar{X}^2 \right]. \tag{B-31}
 \end{aligned}$$

From equations B-14 and B-29, we have

$$\begin{aligned}
 b &= \sum_{t=1}^n w_t Y_t \\
 &= \sum_{t=1}^n H (X_t - \bar{X}) Y_t \\
 &= H \sum_{t=1}^n (X_t Y_t - \bar{X} Y_t) \\
 &= H \left[\sum_{t=1}^n X_t Y_t - n \bar{X} \bar{Y} \right] \tag{B-32}
 \end{aligned}$$

Finally, the "best linear unbiased" estimator of the parameter b can be obtained by substituting equation B-31 into B-32:

$$\hat{b} = \left[\sum_{t=1}^n X_t Y_t - n \bar{X} \bar{Y} \right] / \left[\sum_{t=1}^n X_t^2 - n \bar{X}^2 \right]. \tag{B-33}$$

Comparing equations B-33 and B-8 shows that the formula for calculating the "best linear unbiased" estimate of the parameter b is exactly the same as the formula used for the "least-squares" estimation.

The "Maximum Likelihood" Criterion

Under the "best linear unbiased" criterion, the probability distribution of the residuals is not specified. If the probability distribution of the

residuals is specified explicitly to be a "normal distribution," the combination of this "normality" assumption with conditions B-1, B-12, and B-13 yields the following likelihood function L:

$$L(a,b,S^2) = (2\pi S^2)^{(-\frac{1}{2}n)} \exp [-(2S^2)^{-1} \sum_{t=1}^n (Y_t - a - bX_t)^2] \quad (B-34)$$

where $\pi = 3.14159\cdots$,

and $\exp = \text{exponential}$.

The above likelihood function gives the probability density at the sample data points (Y_t, X_t) . For a given set of the data points, the function L becomes a function of the parameters a, b, and S^2 . Consequently, the problem of finding a "maximum likelihood" estimator of the parameters a, b, and S^2 is the problem of finding an estimate of the parameters that maximizes the probability of obtaining the given observed data points.

The function $L(a,b,S^2)$ can be maximized by setting the partial derivative of $\ln L$, with respect to the parameters a, b, and S^2 , equal to zero and solving the resulting equations for the parameters a, b, and S^2 :

$$\ln L = -\frac{1}{2}n \ln 2\pi - \frac{1}{2}n \ln S^2 - (2S^2)^{-1} \sum_{t=1}^n (Y_t - a - bX_t)^2 \quad (B-35)$$

where $\ln = \text{natural logarithm}$.

$$\frac{\partial \ln L}{\partial a} = (S^2)^{-1} \sum_{t=1}^n (Y_t - a - bX_t) = 0. \quad (B-36)$$

$$\frac{\partial \ln L}{\partial b} = (S^2)^{-1} \sum_{t=1}^n (Y_t - a - bX_t)X_t = 0. \quad (B-37)$$

$$\frac{\partial \ln L}{\partial S^2} = -n(2S^2)^{-1} + (2S^4)^{-1} \sum_{t=1}^n (Y_t - a - bX_t)^2 = 0. \quad (B-38)$$

Comparing equations B-5 and B-6 with equations B-36 and B-37 indicates that the "maximum likelihood" estimator is a "least-squares" estimator. Similarly, it can be easily shown that the "maximum likelihood" estimator is also a "best linear unbiased" estimator. In summary, the following important properties and implications are established from the preceding derivations:

1. Under the strict "least-squares" criterion, it is not necessary to assume that the residuals are statistically "random variables." Without this statistical assumption, the "least-squares" method is simply a mathematical method of fitting a linear equation to a set of observed data points (Y_t, X_t) so that the sum of the squared distances from the data points to the fitted equation will be minimized.

2. Under the "best linear unbiased" criterion, statistical assumptions regarding the residuals are required. Given the statistical assumptions B-12 and B-13, the "best linear unbiased" estimator is also a "least-squares" estimator. On the other hand, excepting the case in which the conditions B-12 and B-13 are imposed upon the residuals, the "least-squares" estimator is not always a "best linear unbiased" estimator. If assumptions B-12 and B-13 are satisfied, the method of "least-squares" provides the best available estimate in the sense that the variance of the estimator is smaller than, or at most equal to, the variances of all other "linear unbiased" estimators, and this holds for any probability distribution of the residuals.

3. Under conditions B-12 and B-13 and with the additional assumption of the "normality" of the residuals, the "maximum likelihood" estimator is a "best linear unbiased" estimator and it is also a "least-squares" estimator. Without the "normality" assumption for the residuals, the "maximum likelihood" estimator is not necessarily a "best linear unbiased" estimator. Hence, if a "best linear unbiased" estimator is desired, it will require the "normality" assumption under the "maximum likelihood" criterion.

APPENDIX C.--ALUMINUM CONSUMPTION DATA USED TO DERIVE STATISTICAL PROJECTIONS
(Thousand short tons, aluminum content)

Year	Metal uses						Nonmetal uses				
	Construc- tion	Transporta- tion	Elec- trical	Cans and containers	Appliances and equipment	Machinery	Dissi- pative	Other metal	Refrac- tories	Chemil- cals	Abra- sives
1960	528	308	234	130	226	156	155	64	NA	NA	NA
1961	572	361	277	146	248	167	155	76	NA	NA	NA
1962	628	450	298	160	286	188	180	130	NA	NA	NA
1963	700	514	316	208	301	212	210	99	NA	NA	NA
1964	770	541	369	237	343	232	250	122	NA	NA	NA
1965	841	648	481	278	390	258	295	111	NA	NA	NA
1966	862	706	580	310	405	297	305	213	156	169	88
1967	834	640	554	364	376	278	370	182	148	169	74
1968	975	709	586	433	444	309	415	183	121	212	69
1969	1,038	713	632	597	481	320	415	162	134	233	76
1970	976	553	610	614	416	272	340	83	136	298	82
1971	1,213	648	632	636	436	292	320	70	141	358	73
1972	1,394	802	688	766	504	345	370	82	174	347	93
1973	1,683	1,016	827	868	600	434	425	44	245	356	123
1974	1,432	880	822	952	520	472	370	50	255	377	141
1975	980	614	538	840	337	294	255	39	195	307	85
1976 ^p	1,302	887	594	1,075	468	412	335	12	197	325	106

^pPreliminary.

NA Not available.

APPENDIX D.--DATA FOR MACROECONOMIC VARIABLES
USED TO DERIVE STATISTICAL PROJECTIONS

Year	U.S. gross national product, billion 1972 dollars	U.S. population, million persons	U.S. gross private domestic investment, billion 1973 dollars	U.S. new construction activity, billion 1973 dollars
1960	736.8	181	117.70	84.27
1961	755.3	184	113.47	86.13
1962	799.1	187	127.77	90.28
1963	830.7	189	133.29	95.76
1964	874.4	192	140.57	98.52
1965	925.9	194	159.43	104.91
1966	981.0	197	171.61	105.31
1967	1,007.7	199	161.74	104.57
1968	1,051.8	201	168.50	111.61
1969	1,078.8	203	178.36	114.55
1970	1,075.3	205	163.06	109.90
1971	1,107.5	207	176.29	121.20
1972	1,171.1	209	199.22	131.29
1973	1,235.0	210	220.00	137.90
1974	1,214.0	212	195.40	125.87
1975	1,191.1	214	152.74	111.67
1976	1,264.7	215.1	190.73	116.67

Year	Federal Reserve Board industrial production indexes (1967=100)				
	Total production	Textile mill products	Paper and products	Chemicals and products	Basic chemicals
1960	66.2	69.3	68.0	56.4	60.9
1961	66.7	71.4	71.8	59.2	63.2
1962	72.2	76.2	76.2	65.7	70.1
1963	76.5	78.9	80.6	71.8	77.3
1964	81.7	85.2	85.4	78.8	85.2
1965	89.8	92.8	91.9	87.8	93.8
1966	97.8	98.4	99.2	95.7	99.6
1967	100.0	100.0	100.0	100.0	100.0
1968	106.3	107.9	107.3	109.5	107.4
1969	111.1	112.6	115.8	118.4	117.0
1970	107.8	111.8	115.2	120.4	117.9
1971	109.6	116.5	120.4	125.9	123.7
1972	119.7	132.7	128.8	143.6	137.7
1973	129.8	142.9	137.4	154.5	147.7
1974	129.3	132.8	134.5	159.4	153.3
1975	117.8	122.3	116.3	147.2	135.9
1976	129.8	135.9	133.1	169.4	158.5

Year	Federal Reserve Board industrial production indexes (1967=100)				
	Synthetic materials	Paints	Petroleum products	Rubber and plastics products	Tires
1960	42.9	72.9	76.7	52.2	76.6
1961	46.3	71.0	79.8	54.5	73.5
1962	54.8	75.1	84.0	61.8	83.5
1963	60.8	82.1	87.9	66.4	86.4
1964	70.8	87.2	90.3	75.3	95.7
1965	85.2	95.3	92.1	86.2	101.8
1966	96.7	100.3	96.1	96.8	109.7
1967	100.0	100.0	100.0	100.0	100.0
1968	126.9	107.0	105.1	119.7	122.1
1969	141.2	112.0	108.4	130.2	124.0
1970	145.1	105.6	113.2	132.3	112.2
1971	160.4	111.1	116.7	147.6	125.7
1972	194.7	118.0	122.1	172.4	141.9
1973	221.1	114.7	128.5	184.1	141.2
1974	224.4	119.2	124.7	195.2	147.9
1975	191.0	114.2	124.1	166.7	124.7
1976	232.6	124.2	132.7	200.2	127.9

Year	Federal Reserve Board industrial production indexes (1967=100)				
	Rubber excluding tires	Plastics products, n.e.c.	Stone, clay, and glass	Iron and steel	Basic steel and mill products
1960	66.0	32.0	79.0	77.7	79.3
1961	68.7	36.6	78.7	74.2	76.1
1962	77.4	41.5	83.5	77.3	78.2
1963	80.8	47.6	88.5	84.3	84.6
1964	88.1	57.0	92.7	95.9	96.3
1965	94.7	72.2	98.3	105.2	105.2
1966	101.6	85.9	101.1	108.4	107.8
1967	100.0	100.0	100.0	100.0	100.0
1968	107.2	126.7	106.3	103.2	103.3
1969	115.2	144.1	109.6	112.6	112.9
1970	107.7	161.4	106.0	104.7	106.5
1971	114.1	183.8	110.9	96.1	95.9
1972	127.6	221.5	120.8	107.1	107.0
1973	144.1	237.5	133.5	122.3	122.9
1974	141.3	260.7	133.1	119.8	121.6
1975	121.5	223.2	117.8	95.8	96.8
1976	143.8	281.9	135.8	104.4	105.4

Year	Federal Reserve Board industrial production indexes (1967=100)				
	Fabricated metal products	Metal cans	Hardware, plumbing, and structural metal	Nonelectrical machinery	Construction and allied equipment
1960	71.1	74.8	69.8	56.9	65.4
1961	69.4	79.5	68.3	55.4	59.0
1962	75.4	78.5	73.0	62.1	65.4
1963	77.8	76.8	76.2	66.3	72.3
1964	82.6	82.3	81.6	75.6	86.6
1965	90.8	86.0	90.0	85.0	97.0
1966	97.2	90.4	96.6	98.8	104.7
1967	100.0	100.0	100.0	100.0	100.0
1968	105.6	110.3	103.4	101.8	104.5
1969	107.9	113.2	106.8	109.3	107.4
1970	102.4	120.9	105.2	104.4	99.0
1971	103.5	114.4	107.9	100.2	100.6
1972	112.1	119.0	118.3	116.0	112.6
1973	124.7	126.5	131.1	133.7	127.8
1974	124.2	132.9	132.8	140.1	137.8
1975	109.9	126.5	120.3	125.1	129.3
1976	123.3	136.7	130.0	134.7	137.9

Year	Federal Reserve Board industrial production indexes (1967=100)				
	Metalworking machinery	Special and general industrial equipment	Electrical machinery	Major electrical equipment and parts	Household appliances
1960	60.5	62.2	51.6	62.1	59.4
1961	56.3	61.5	54.6	61.9	61.8
1962	66.7	67.3	62.9	67.1	68.1
1963	67.6	71.5	64.7	68.8	79.9
1964	76.5	80.9	68.4	75.5	82.8
1965	86.3	91.6	81.7	86.2	92.7
1966	98.9	102.7	97.9	99.5	99.2
1967	100.0	100.0	100.0	100.0	100.0
1968	94.2	98.5	105.5	101.0	108.7
1969	93.9	102.8	111.9	106.8	115.2
1970	85.6	95.6	108.1	105.0	117.4
1971	74.1	88.4	107.7	98.6	124.8
1972	85.7	96.9	122.2	113.5	145.4
1973	101.0	115.2	143.1	131.3	155.8
1974	104.2	118.3	143.8	135.7	146.3
1975	93.0	104.1	116.5	106.1	117.8
1976	98.3	111.2	131.7	114.5	128.4

Year	Federal Reserve Board industrial production indexes (1967=100)				
	Communication equipment	Transportation equipment	Motor vehicles and parts	Automobiles	Trucks, buses, and trailers
1960	56.1	65.4	74.7	75.4	65.8
1961	66.3	61.5	65.5	64.0	64.4
1962	80.6	71.1	79.8	82.1	71.9
1963	77.6	78.0	88.3	91.4	85.8
1964	74.2	80.0	90.7	92.9	90.7
1965	81.6	95.1	115.9	121.0	111.3
1966	92.5	102.0	113.9	116.1	114.4
1967	100.0	100.0	100.0	100.0	100.0
1968	109.5	111.1	120.3	120.9	122.6
1969	111.5	108.4	116.5	113.8	123.7
1970	105.8	89.5	92.3	86.6	102.0
1971	97.5	97.9	118.6	116.0	123.4
1972	101.4	108.2	135.8	128.6	154.2
1973	108.8	118.3	148.8	138.3	181.5
1974	114.1	108.7	128.2	107.9	172.4
1975	105.6	97.4	111.1	101.1	140.6
1976	107.7	110.6	140.7	132.0	NA ¹

¹The trucks, buses, and trailers index number is not available after 1975 because the classification has been changed.

Year	Federal Reserve Board industrial production indexes (1967=100)			
	Aircraft and parts	Ships and boats	Food and products	Ordinance
1960	55.9	63.7	78.6	50.1
1961	58.3	65.8	80.9	42.3
1962	64.0	68.2	83.4	48.6
1963	68.8	71.3	86.4	55.7
1964	67.7	77.3	90.4	54.9
1965	71.9	86.0	92.4	66.7
1966	88.4	94.1	96.0	82.9
1967	100.0	99.9	100.0	100.0
1968	103.0	97.6	102.6	115.6
1969	97.3	98.7	106.1	111.9
1970	81.1	94.7	108.9	92.7
1971	67.7	95.3	112.8	85.1
1972	67.0	114.1	116.8	84.2
1973	75.1	120.7	120.9	81.3
1974	78.3	129.4	124.0	78.9
1975	73.3	133.6	123.4	76.6
1976	69.3	151.3	132.3	72.7

APPENDIX E.--MACROECONOMIC EXPLANATORY VARIABLES USED IN ESTIMATED ALUMINUM
END-USE CONSUMPTION EQUATIONS AND ASSOCIATED R^2 VALUES

The number shown after each macroeconomic variable is the R^2 value;
* indicates the best R^2 value.

Construction

GNP 0.8280
U.S. population 0.7501
*New construction activity 0.9549
FRB index--Total production 0.8519
FRB index--Construction and allied equipment 0.7205
FRB index--Hardware, plumbing, and structural metal 0.8557

Transportation

GNP 0.7776
FRB index--Total production 0.8197
FRB index--Tires 0.8078
FRB index--Automobiles 0.8172
FRB index--Transportation equipment 0.8796
FRB index--Petroleum products 0.7135
*FRB index--Vehicles 0.9272
FRB index--Trucks, buses, and trailers 0.8941
FRB index--Aircraft and parts 0.1554
FRB index--Ships and boats 0.6762

Electrical

GNP 0.8260
FRB index--Total production 0.8771
FRB index--Communication equipment 0.7993
*FRB index--Major electrical equipment and parts 0.9562
FRB index--Electrical machinery 0.9235
FRB index--Household appliances 0.9257
FRB index--Fabricated metal products 0.8899

Cans and containers

GNP 0.9271
FRB index--Total production 0.9089
FRB index--Metalworking machinery 0.4706
FRB index--Fabricated metal products 0.8778.
*FRB index--Metal cans 0.9473

Appliances and equipment

GNP 0.7404
U.S. population 0.6436
*Gross private domestic investment (GPDI) 0.9647
FRB index--Total production 0.7974
FRB index--Major electrical equipment and parts 0.8491
FRB index--Electrical machinery 0.8300
FRB index--Household appliances 0.8757
FRB index--Communication equipment 0.7009

Machinery

GNP 0.8373
 U.S. population 0.7606
 FRB index--Total production 0.8844
 *FRB index--Nonelectrical machinery 0.8986
 FRB index--Metalworking machinery 0.7098
 FRB index--Special and general industrial machinery 0.8601

Dissipative

GNP 0.6249
 U.S. population 0.5443
 *Gross private domestic investment (GPDI) 0.7827
 FRB index--Total production 0.6836
 FRB index--Chemicals and products 0.5267
 FRB index--Basic chemicals 0.5572

Other Metal

GNP 0.1135
 U.S. population 0.1271
 GPDI 0.0231
 FRB index--Total production 0.0893
 FRB index--Paints 0.0516
 *FRB index--Chemicals and products 0.1932
 FRB index--Basic chemicals 0.1731

Refractories

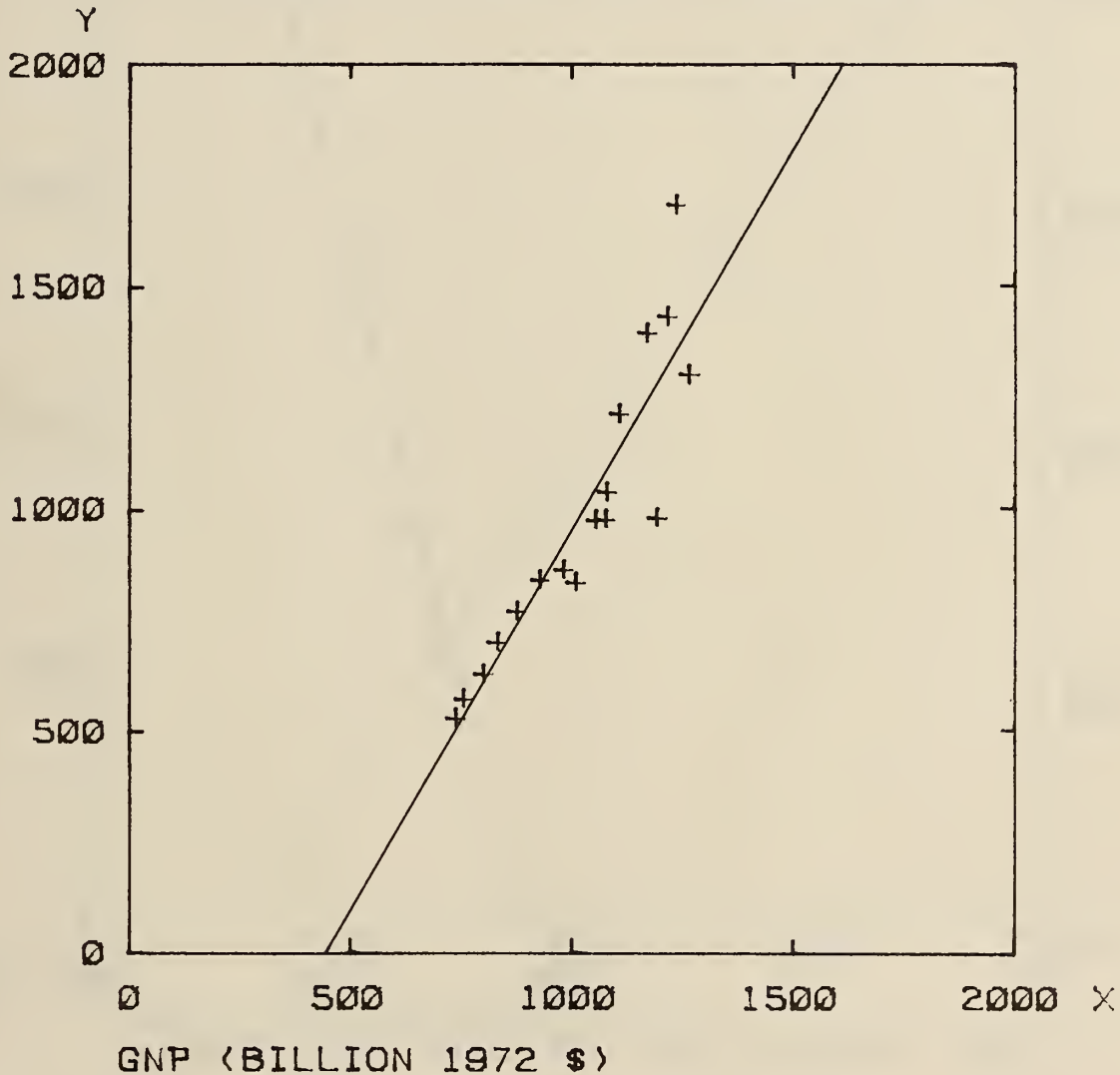
GNP 0.6014
 U.S. population 0.4693
 GPDI 0.4241
 FRB index--Total production 0.6840
 *FRB index--Stone, clay, and glass 0.7288
 FRB index--Iron and steel 0.3764
 FRB index--Basic steel and mill products 0.4083

Chemicals

GNP 0.7079
 U.S. population 0.7181
 GPDI 0.3319
 FRB index--Total production 0.6727
 FRB index--Basic chemicals 0.7406
 FRB index--Chemicals and products 0.7185
 FRB index--Plastics 0.7816
 *FRB index--Paper 0.8248
 FRB index--Textiles 0.7448

Abrasives

GNP 0.4932
 U.S. population 0.3406
 GPDI 0.5201
 *FRB index--Total production 0.6393
 FRB index--Iron and steel 0.5707
 FRB index--Metalworking machinery 0.3077
 FRB index--Basic steel 0.6114

APPENDIX F.--OTHER COMPUTER GRAPHS WITH LOWER R^2 VALUESALUMINUM DEMAND-CONSTRUCTION
THOUSAND SHORT TONS (1960-1976)

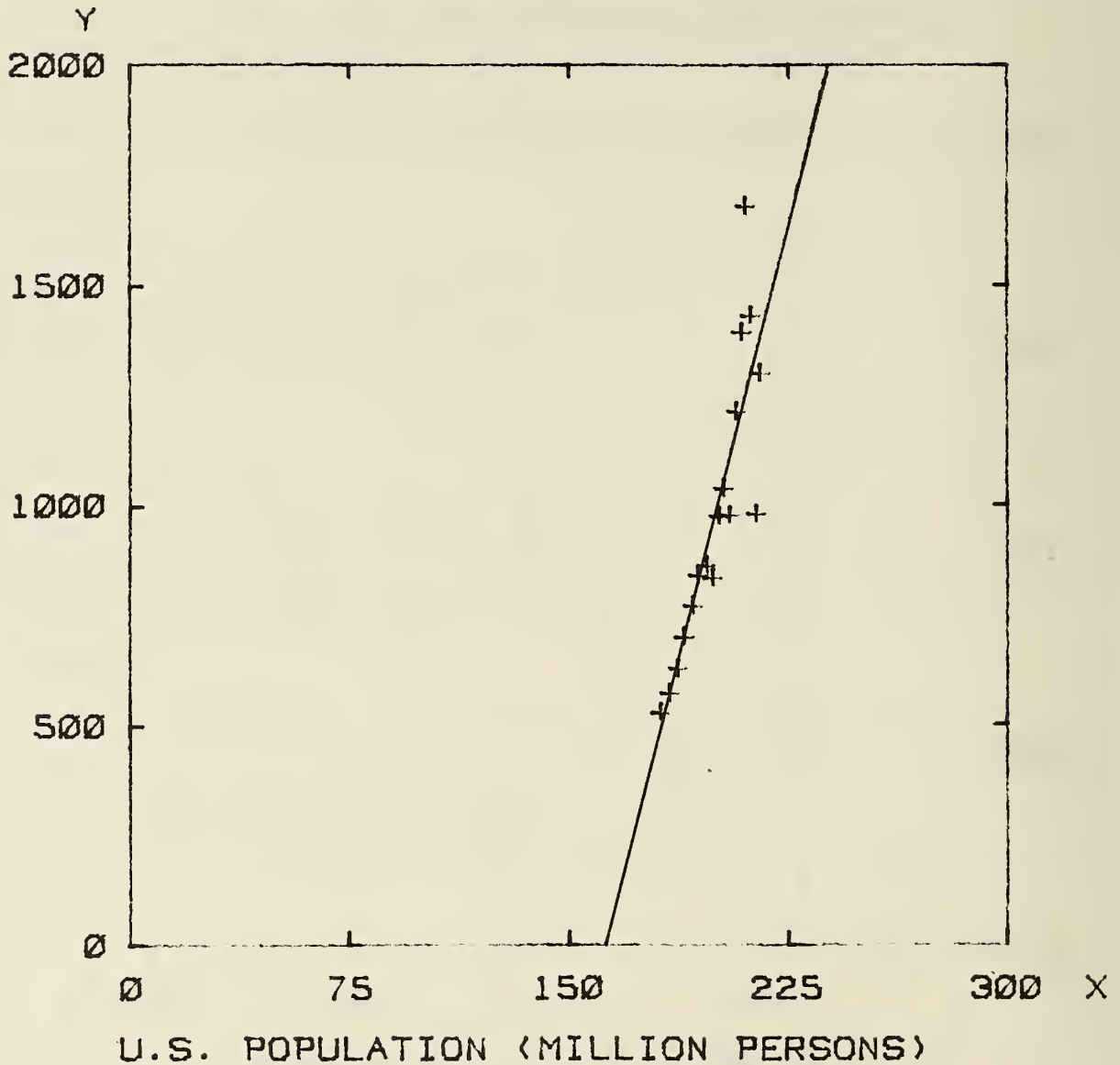
$$Y = -764.6077 + 1.7182 X$$

$$R\text{-SQUARED} = 0.8280$$

PROJECTIONS	Y	X
1985	2277.5807	1770.5000
2000	4032.7886	2792.0000

FIGURE F-1. - Scatter diagram of aluminum demand for construction use and gross national product (GNP).

ALUMINUM DEMAND--CONSTRUCTION
THOUSAND SHORT TONS (1960-1976)



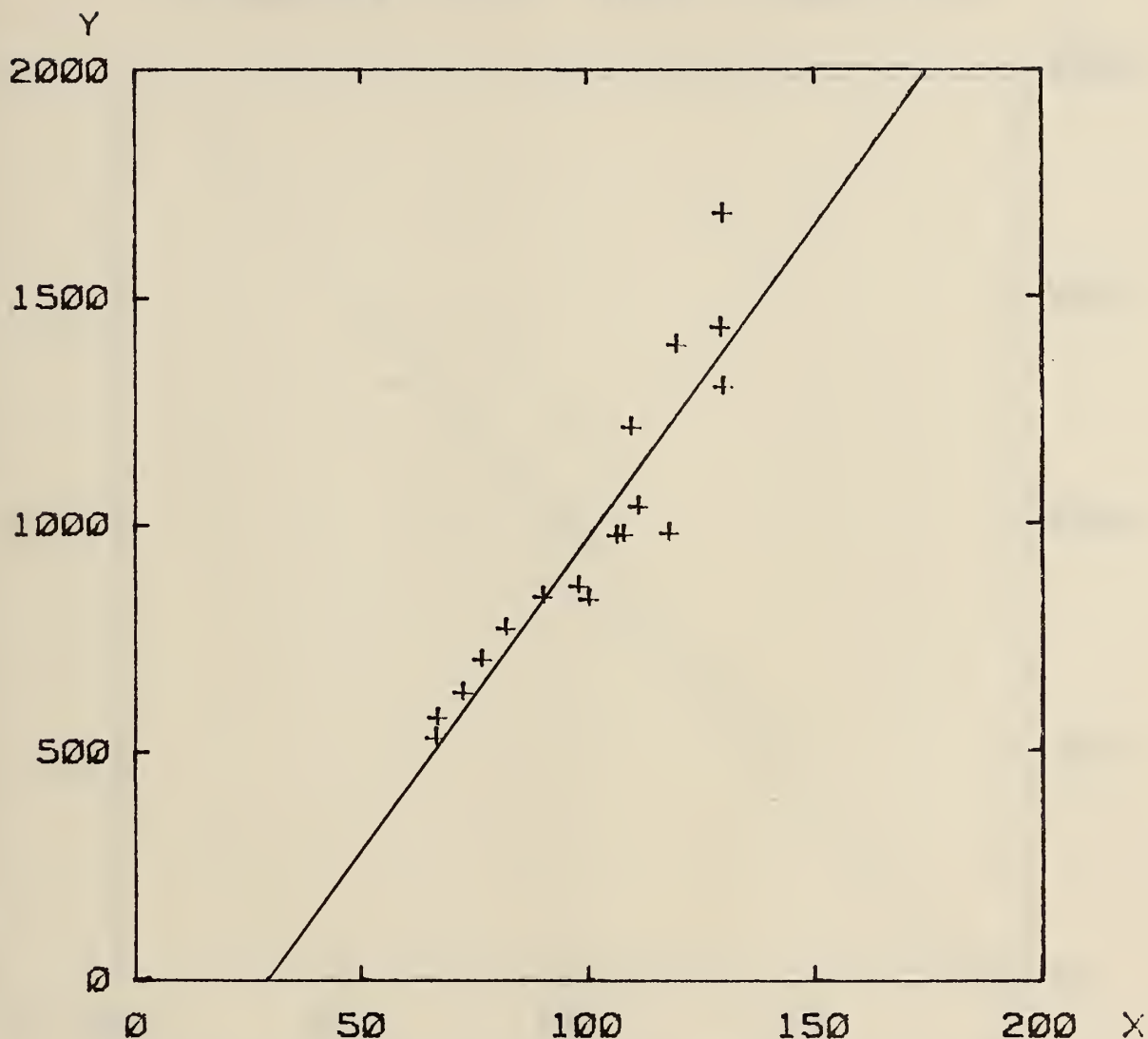
$$Y = -4270.0455 + 26.2771 X$$

$$R\text{-SQUARED} = 0.7501$$

PROJECTIONS	Y	X
1985	1881.4430	234.1000
2000	2627.7151	262.5000

FIGURE F-2. - Scatter diagram of aluminum demand for construction use and U.S. population.

ALUMINUM DEMAND--CONSTRUCTION
THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION
1967 = 100

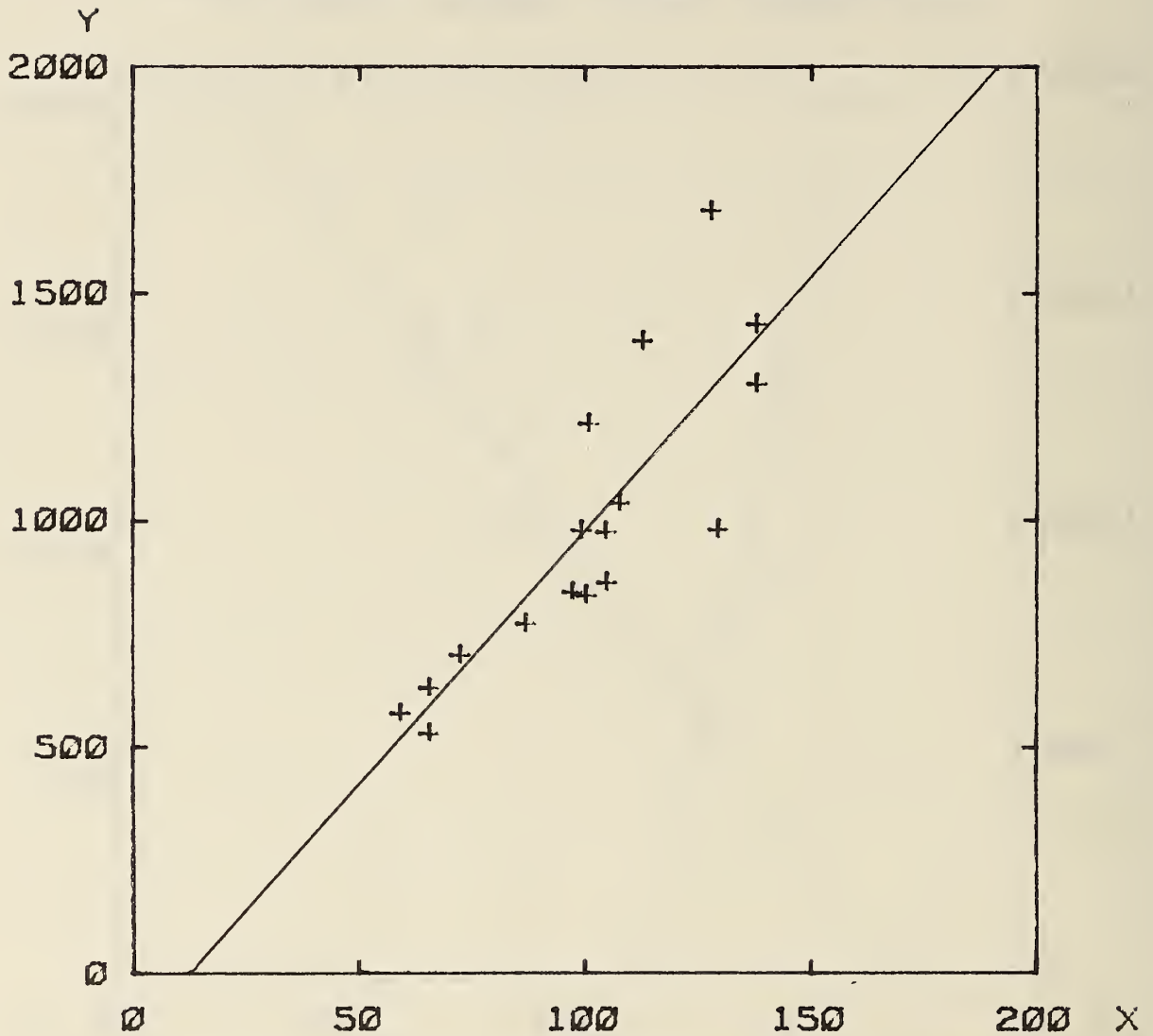
$$Y = - 401.3510 + 13.7556 X$$

$$R\text{-SQUARED} = 0.8519$$

PROJECTIONS	Y	X
1985	2161.3177	186.3000
2000	3862.8858	310.0000

FIGURE F-3. - Scatter diagram of aluminum demand for construction use and FRB index for total production.

ALUMINUM DEMAND--CONSTRUCTION
THOUSAND SHORT TONS (1960-1976)



FRBI-CONSTRUCTION & ALLIED EQUIPMENT
1967 = 100

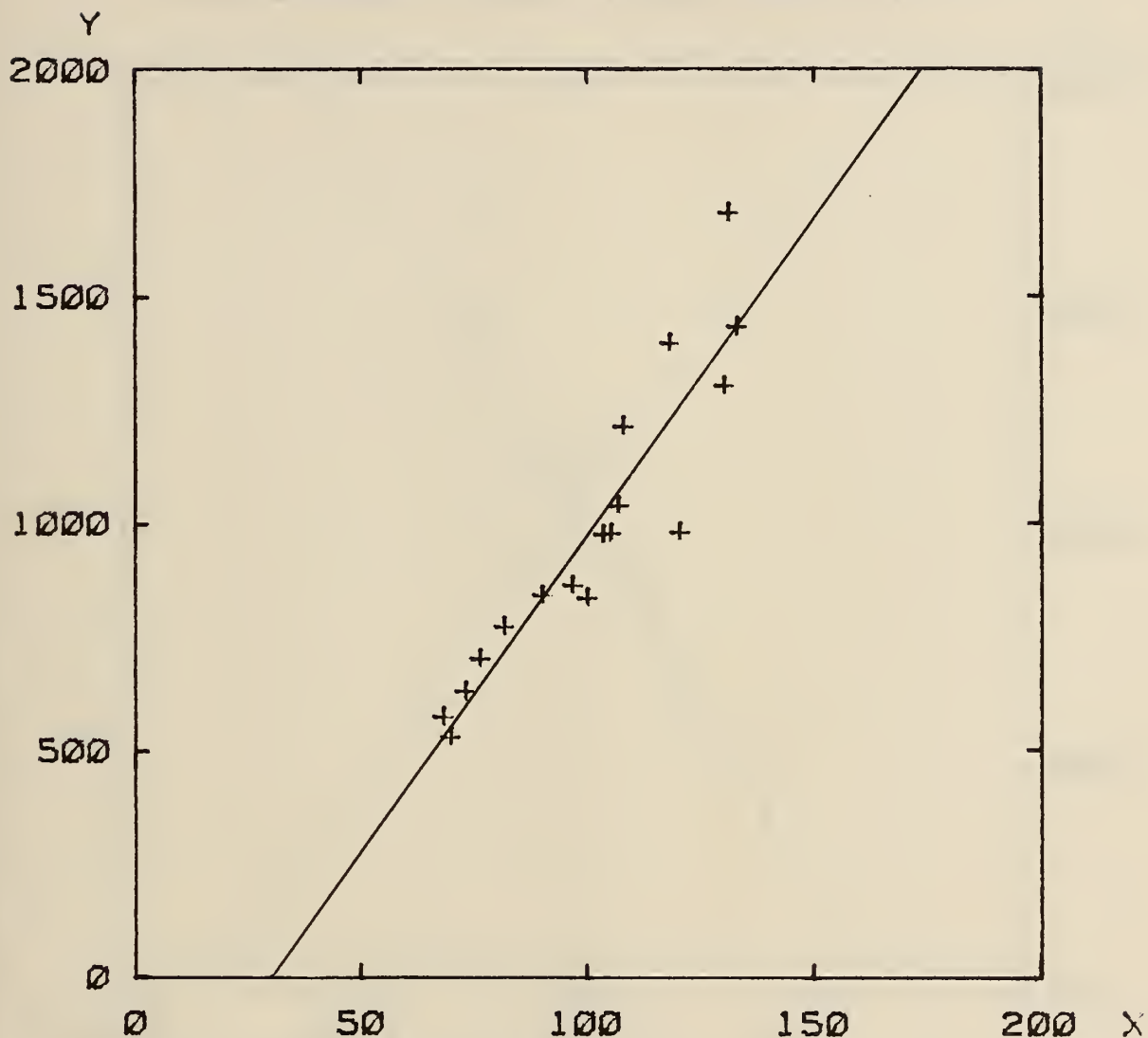
$$Y = -140.7349 + 11.1992 X$$

$$R\text{-SQUARED} = 0.7205$$

PROJECTIONS	Y	X
1985	1974.8049	188.9000
2000	2699.3969	253.6000

FIGURE F-4. - Scatter diagram of aluminum demand for construction use and FRB index for construction and allied equipment.

ALUMINUM DEMAND--CONSTRUCTION
THOUSAND SHORT TONS (1960-1976)



FRBI-HARDWARE, PLUMBING, STRUCTURAL METAL
1967 = 100

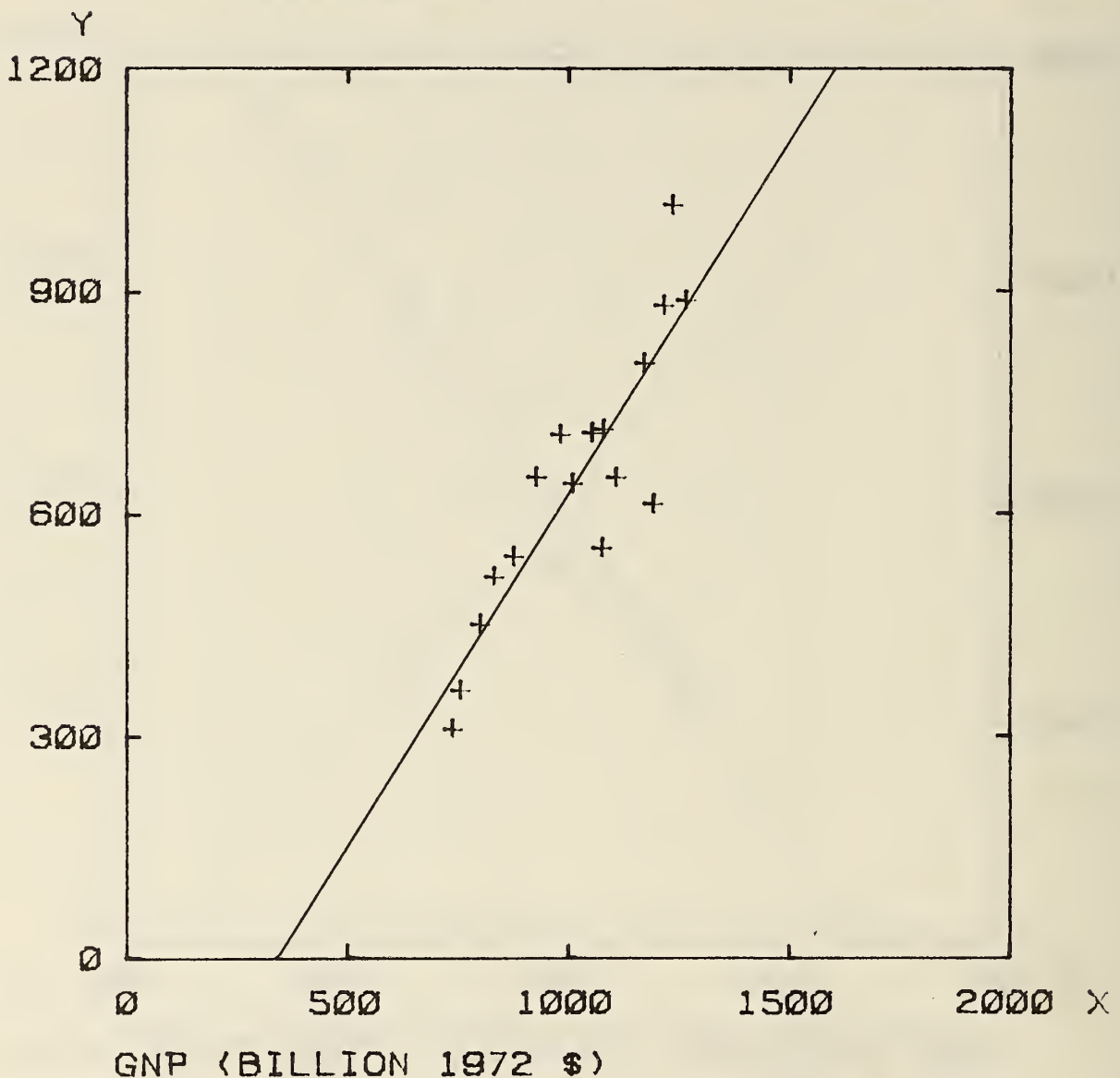
$$Y = -419.8344 + 13.9456 X$$

$$R\text{-SQUARED} = 0.8557$$

PROJECTIONS	Y	X
1985	2214.4983	188.9000
2000	3401.2728	274.0000

FIGURE F-5. - Scatter diagram of aluminum demand for construction use and FRB index for hardware, plumbing, structural metal.

ALUMINUM DEMAND-TRANSPORTATION
THOUSAND SHORT TONS (1960-1976)



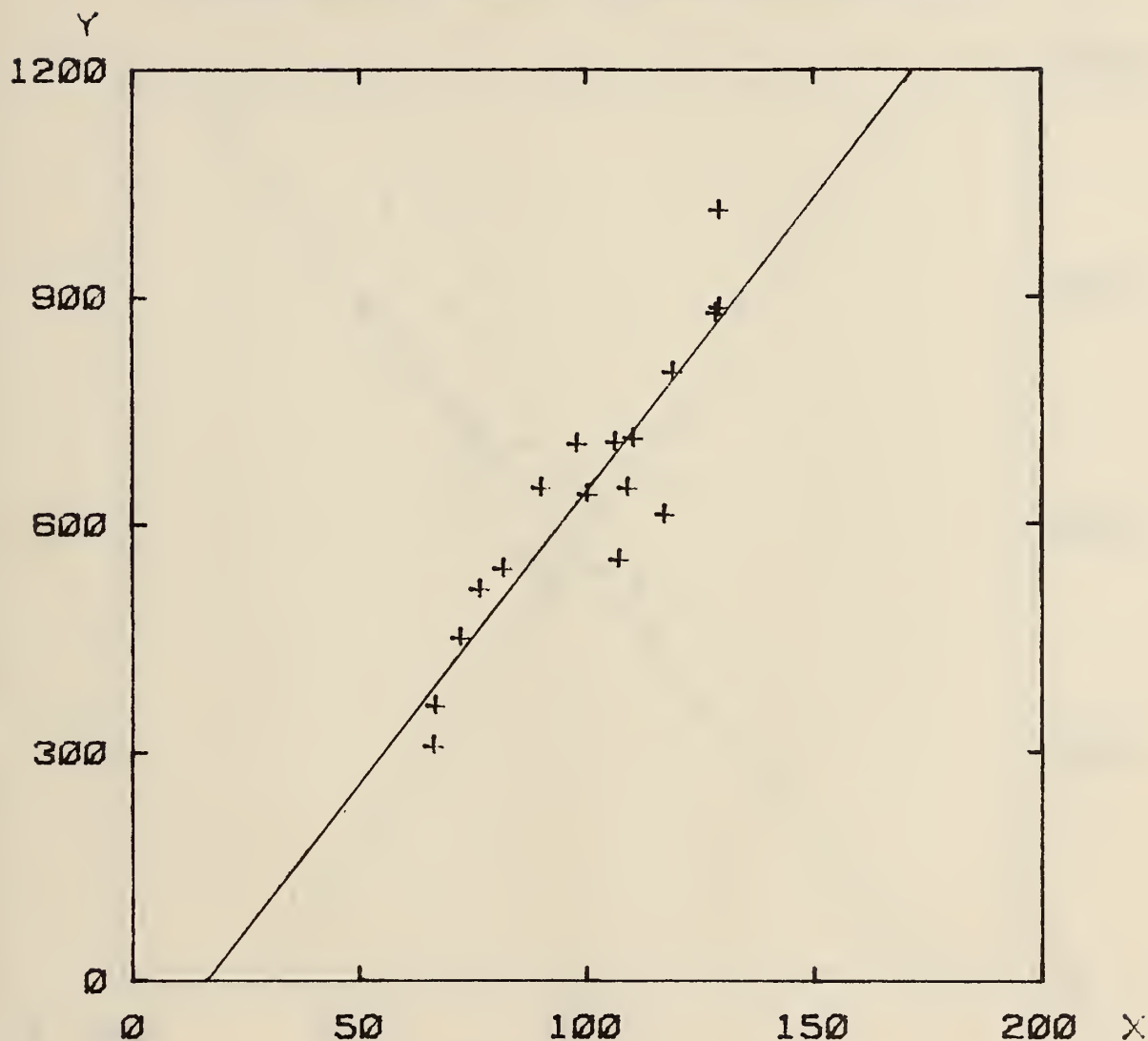
$$Y = -323.7238 + 0.9533 X$$

$$R\text{-SQUARED} = 0.7776$$

PROJECTIONS	Y	X
1985	1364.1987	1770.5000
2000	2338.0552	2792.0000

FIGURE F-6. - Scatter diagram of aluminum demand for transportation use and gross national product (GNP).

ALUMINUM DEMAND--TRANSPORTATION THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION
1967 = 100

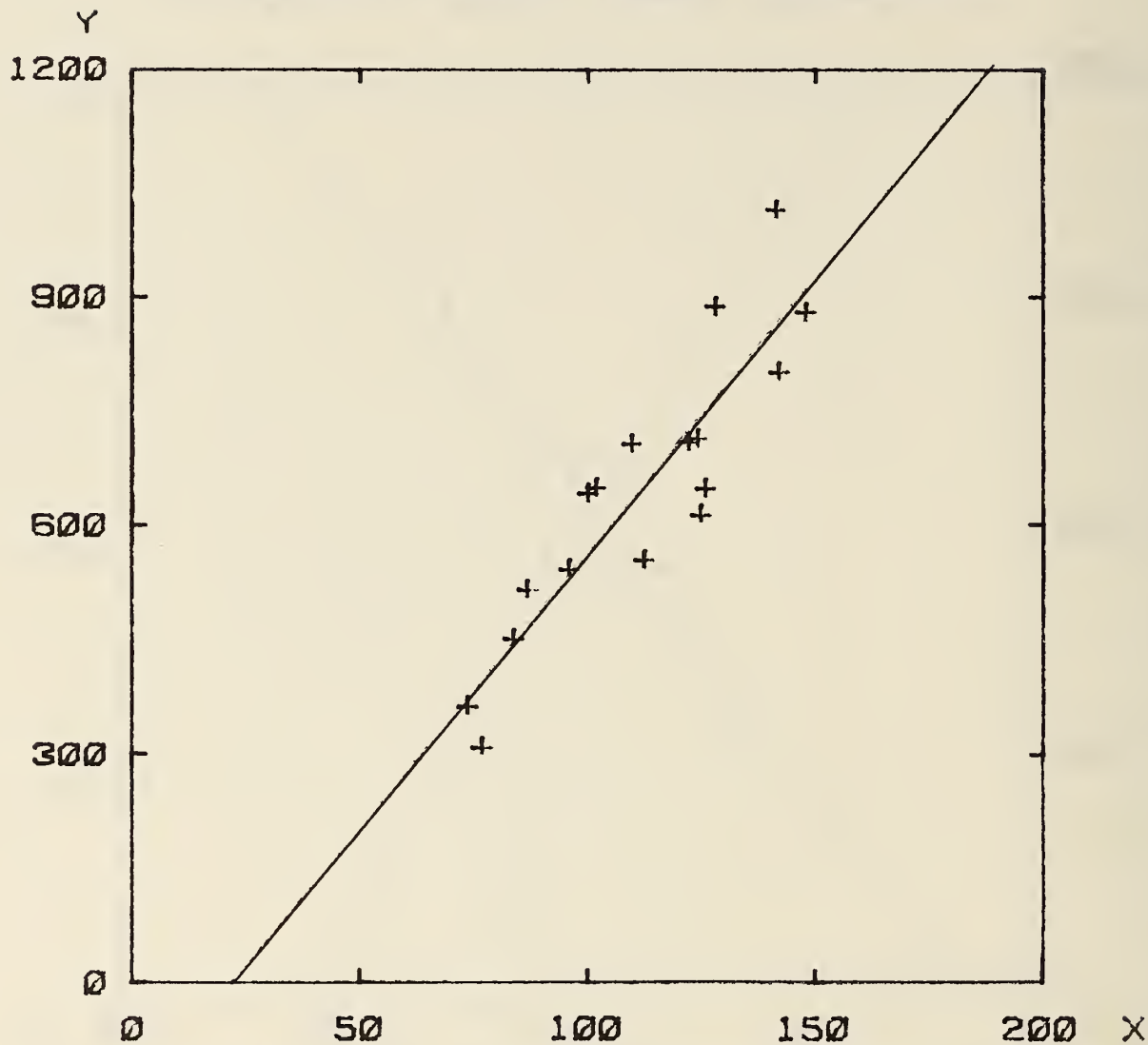
$$Y = -131.5764 + 7.7254 X$$

$$R\text{-SQUARED} = 0.8197$$

PROJECTIONS	Y	X
1985	1307.6810	186.3000
2000	2263.3233	310.0000

FIGURE F-7. - Scatter diagram of aluminum demand for transportation use and FRB index for total production.

ALUMINUM DEMAND--TRANSPORTATION
THOUSAND SHORT TONS (1960-1976)



FRBI-TIRES
1967 = 100

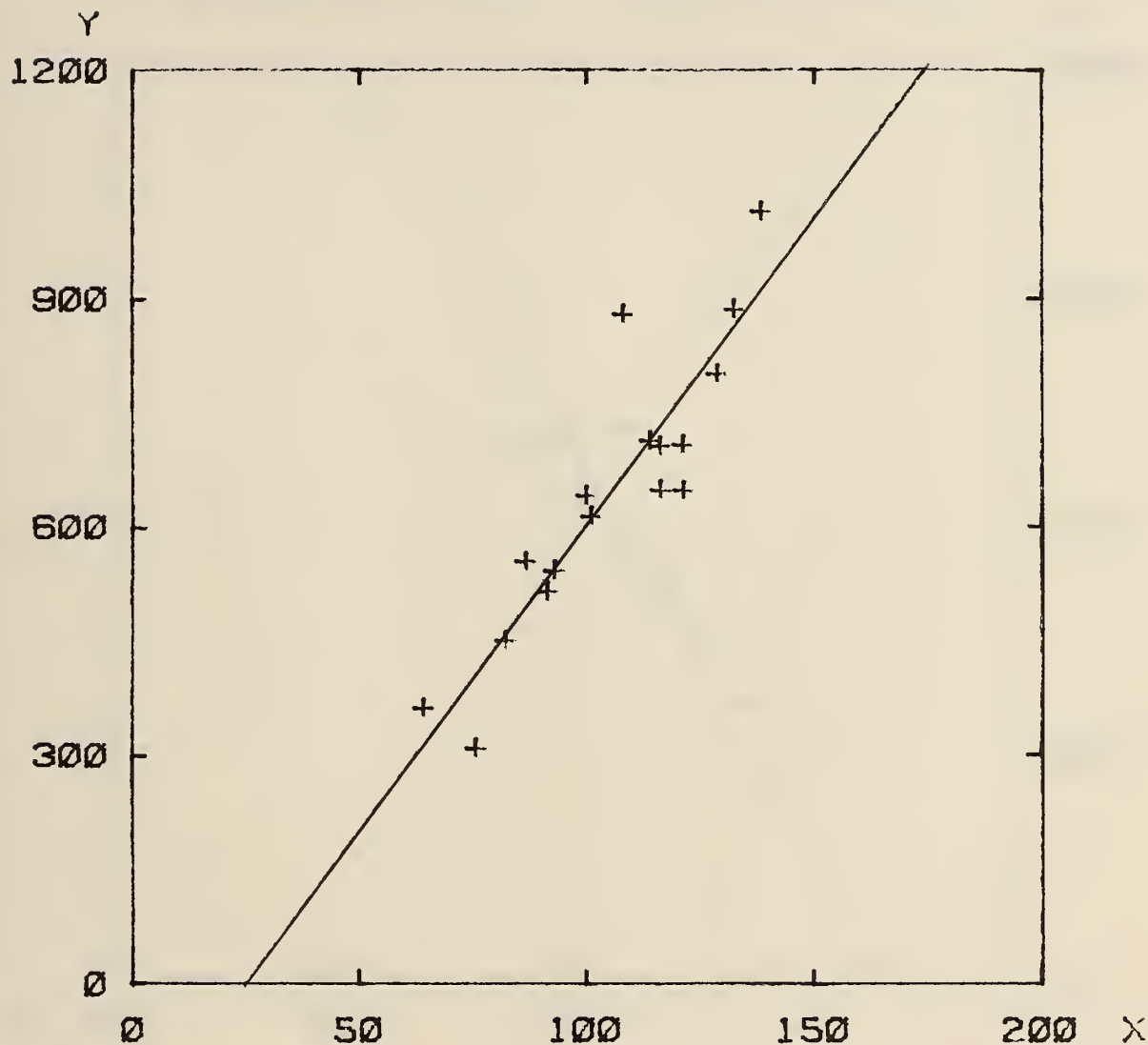
$$Y = -160.6120 + 7.2410 X$$

$$R\text{-SQUARED} = 0.8078$$

PROJECTIONS	Y	X
1985	1175.3678	184.5000
2000	1708.3115	258.1000

FIGURE F-8. - Scatter diagram of aluminum demand for transportation use and FRB index for tires.

ALUMINUM DEMAND--TRANSPORTATION
THOUSAND SHORT TONS (1960-1976)



FRBI-AUTOMOBILES
1967 = 100

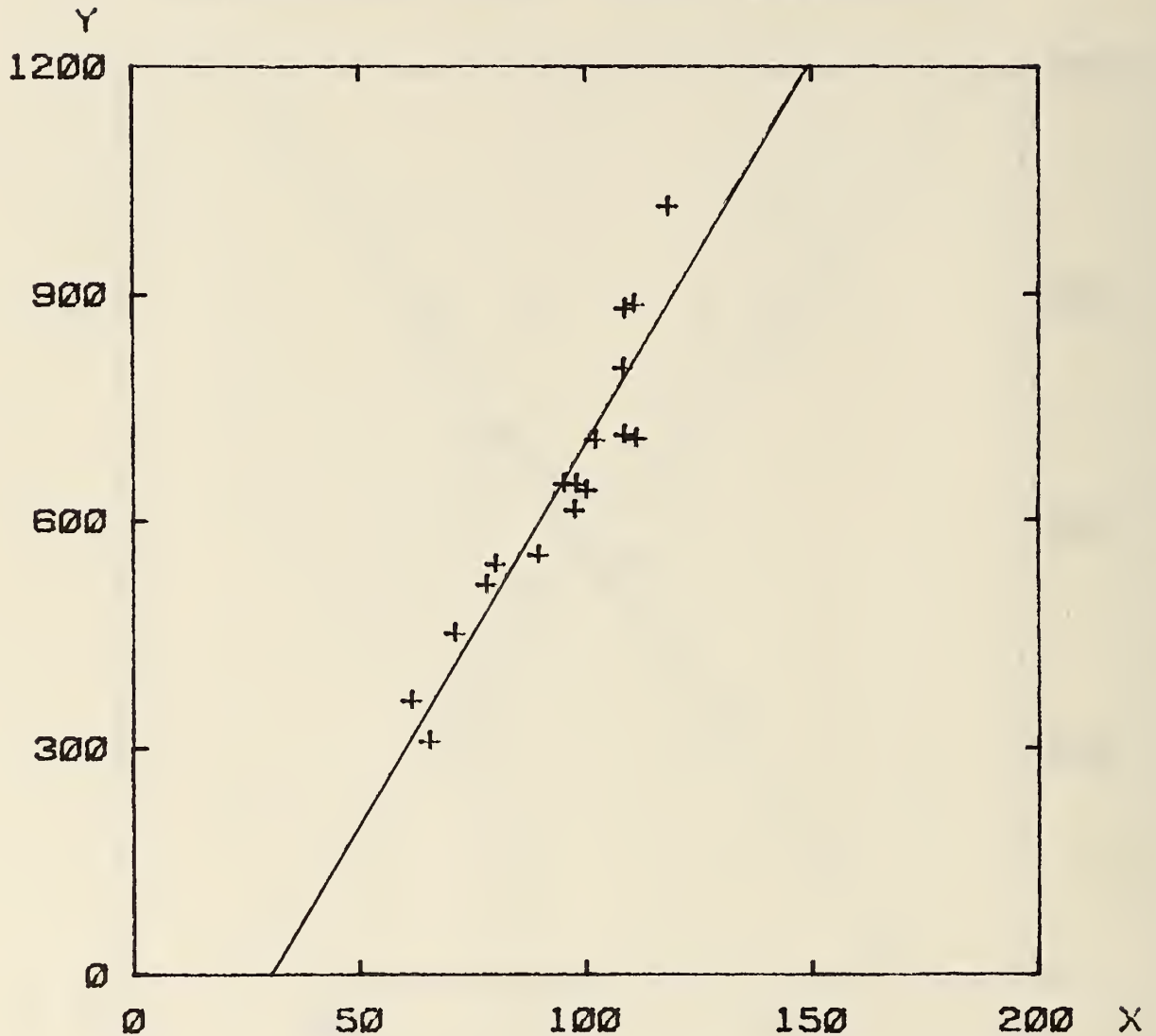
$$Y = - 201.1124 + 8.0582 X$$

$$R-SQUARED = 0.8172$$

PROJECTIONS	Y	X
1985	1008.4270	150.1000
2000	1402.4743	199.0000

FIGURE F-9. - Scatter diagram of aluminum demand for transportation use and FRB index for automobiles.

ALUMINUM DEMAND--TRANSPORTATION
THOUSAND SHORT TONS (1960-1976)



FRB-TRANSPORTATION EQUIPMENT
1967 = 100

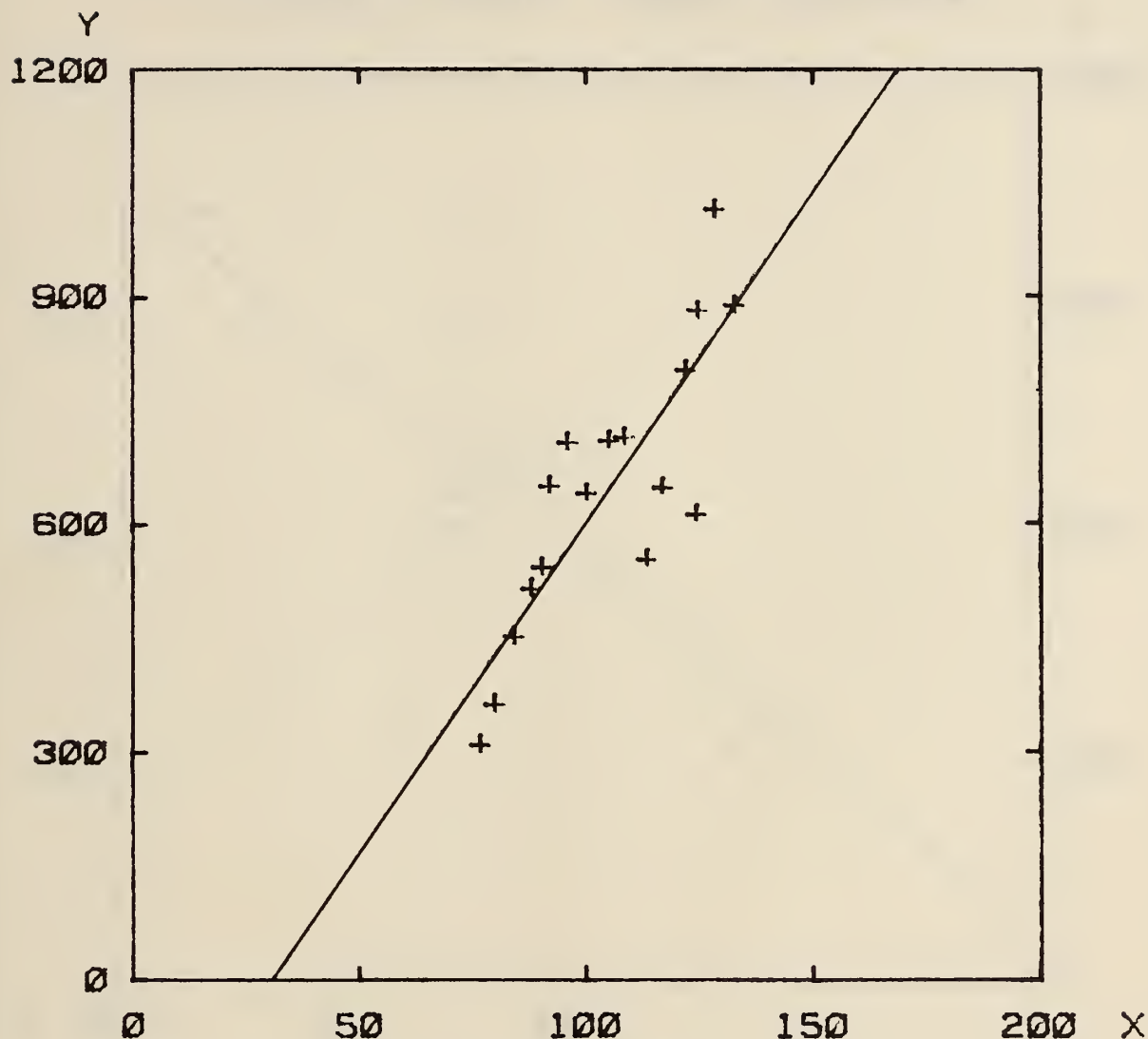
$$Y = - 309.2435 + 10.1341 X$$

$$R\text{-SQUARED} = 0.8796$$

PROJECTIONS	Y	X
1985	1227.1003	151.6000
2000	2629.6728	290.0000

FIGURE F-10. - Scatter diagram of aluminum demand for transportation use and FRB index for transportation equipment.

ALUMINUM DEMAND-TRANSPORTATION THOUSAND SHORT TONS (1960-1976)



FRBI-PETROLEUM PRODUCTS
1967 = 100

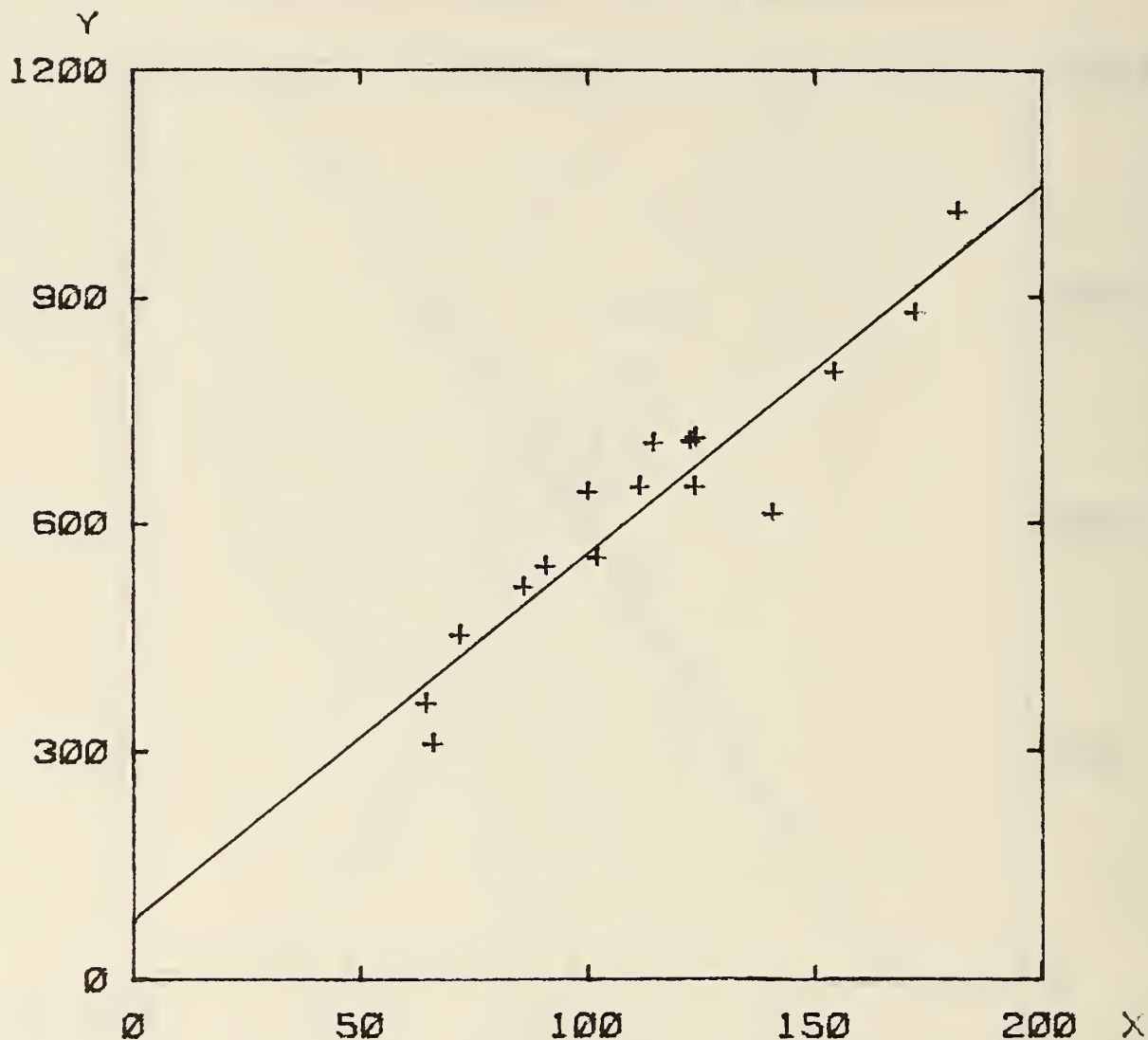
$$Y = -267.9487 + 8.7214 X$$

$$R\text{-SQUARED} = 0.7135$$

PROJECTIONS	Y	X
1985	1306.2746	180.5000
2000	2409.5381	307.0000

FIGURE F-11. - Scatter diagram of aluminum demand for transportation use and FRB index for petroleum products.

ALUMINUM DEMAND--TRANSPORTATION THOUSAND SHORT TONS



FRBI-TRUCKS, BUSES, & TRAILERS
1967 = 100

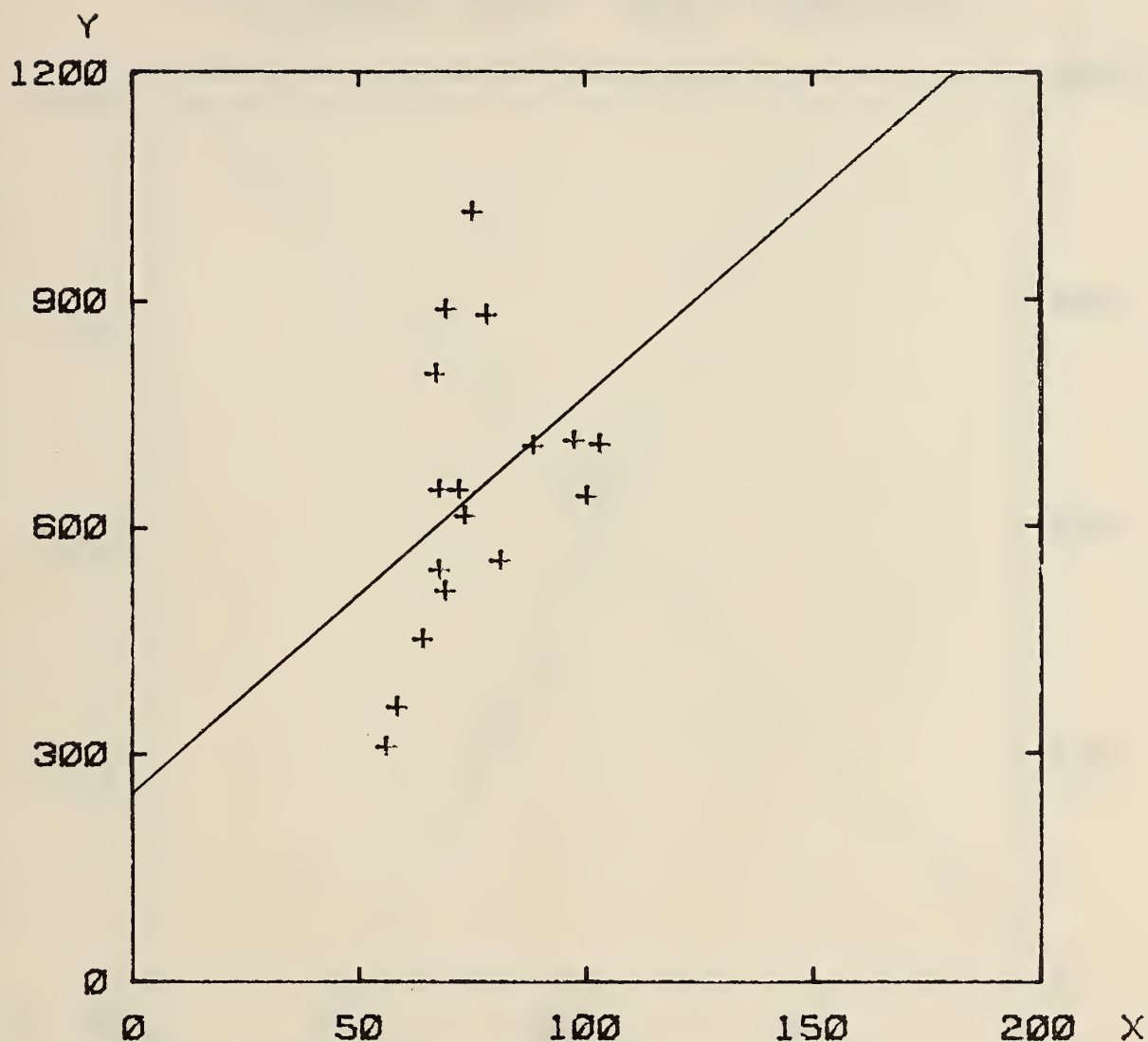
$$Y = 76.9576 + 4.8619 X$$

$$R\text{-SQUARED} = 0.8941$$

PROJECTIONS	Y	X
1985	1309.4727	253.5000
2000	2177.3385	432.0000

FIGURE F-12. - Scatter diagram of aluminum demand for transportation use and FRB index for trucks, buses, and trailers.

ALUMINUM DEMAND--TRANSPORTATION
THOUSAND SHORT TONS (1960-1978)



FRBI-AIRCRAFT & PARTS
1967 = 100

$$Y = 248.7405 + 5.2532 X$$

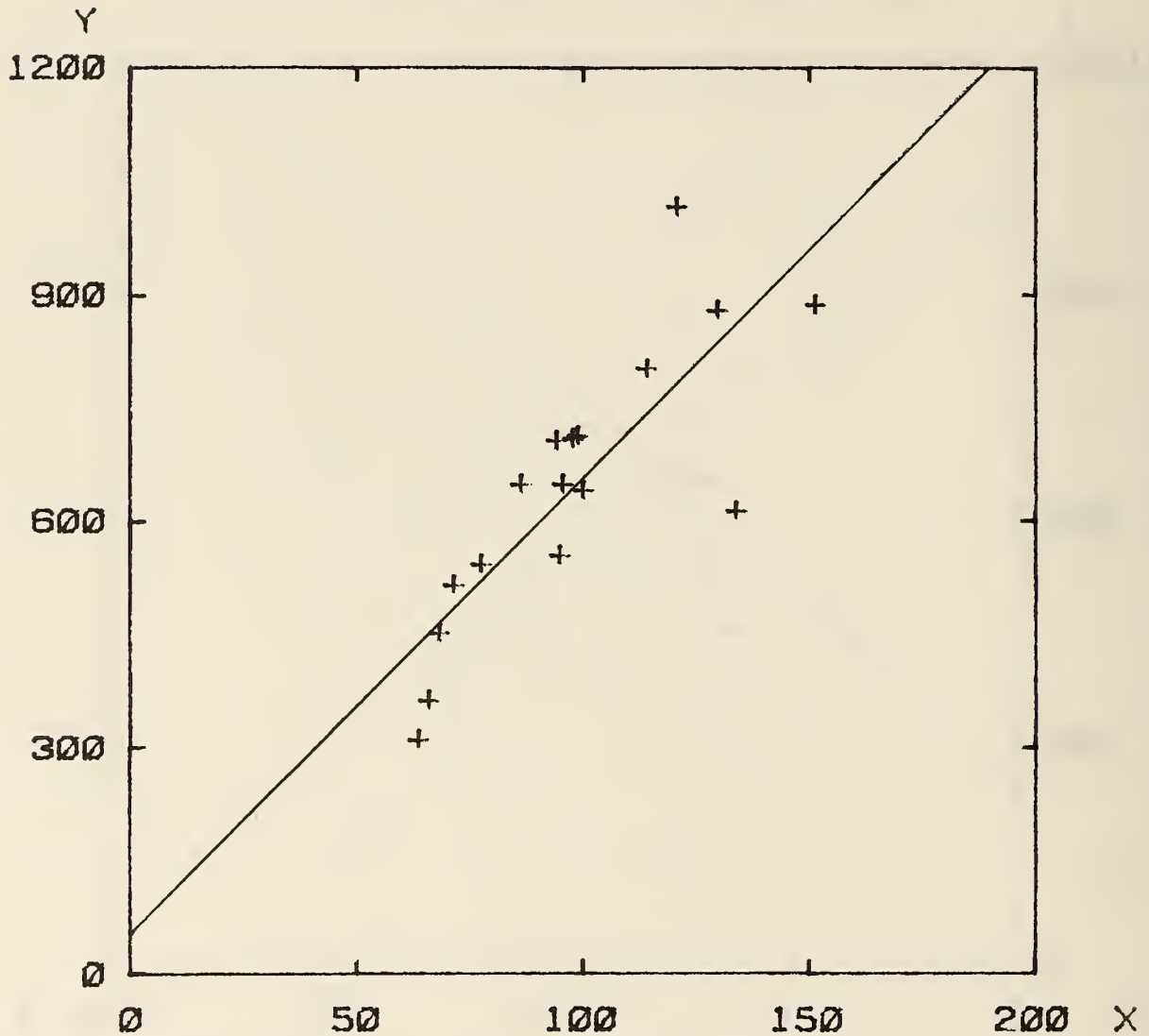
$$R\text{-SQUARED} = 0.1554$$

PROJECTIONS

	Y	X
1985	857.5879	115.9000
2000	1315.6681	203.1000

FIGURE F-13. - Scatter diagram of aluminum demand for transportation use and FRB index for aircraft and parts.

ALUMINUM DEMAND--TRANSPORTATION
THOUSAND SHORT TONS (1960-1976)



FRBI-SHIPS & BOATS
1967 = 100

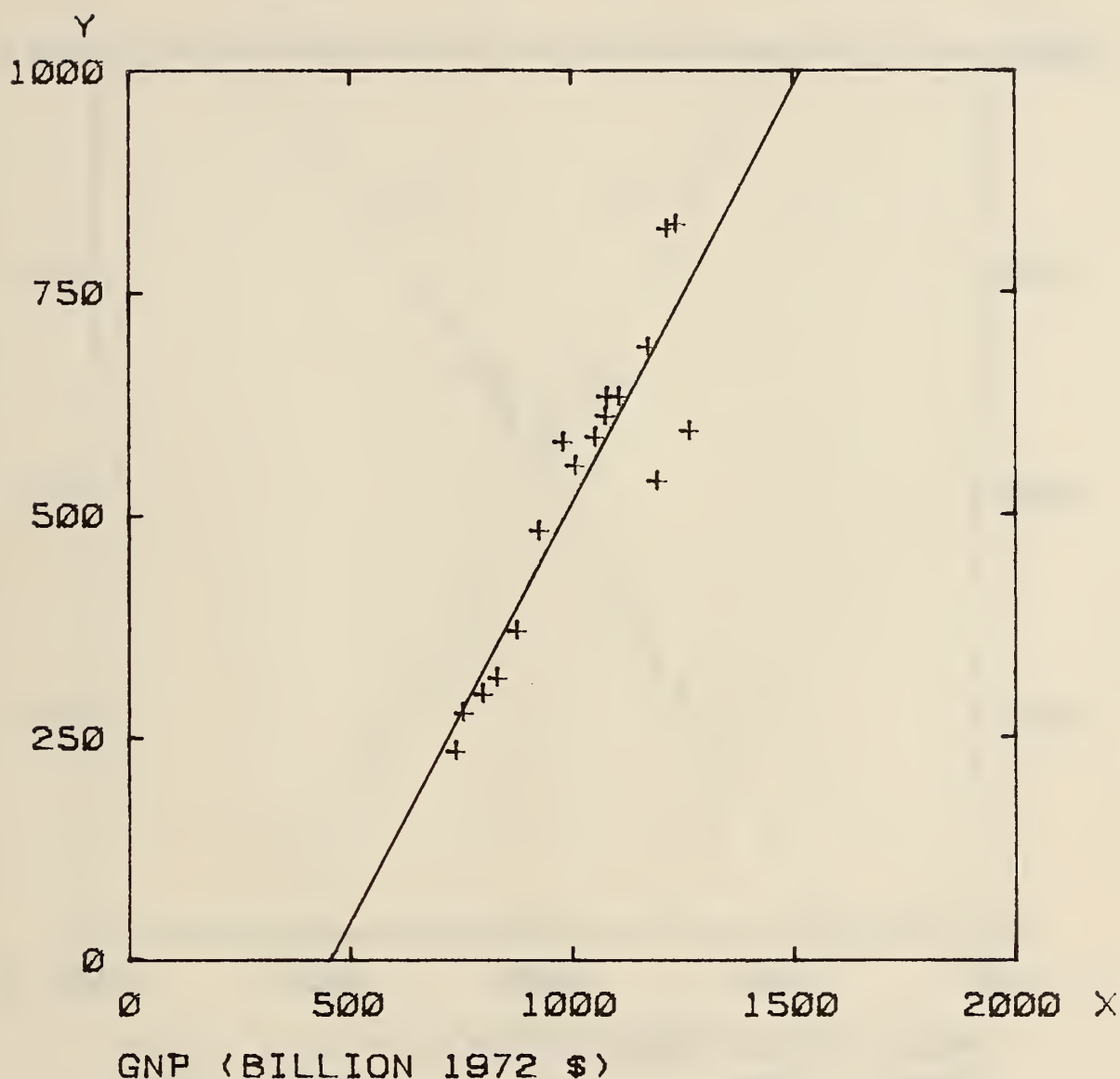
$$Y = 54.5074 + 6.0560 X$$

$$R-SQUARED = 0.6762$$

PROJECTIONS	Y	X
1985	878.1331	136.0000
2000	921.7368	143.2000

FIGURE F-14. - Scatter diagram of aluminum demand for transportation use and FRB index for ships and boats.

ALUMINUM DEMAND-ELECTRICAL
THOUSAND SHORT TONS (1960-1976)



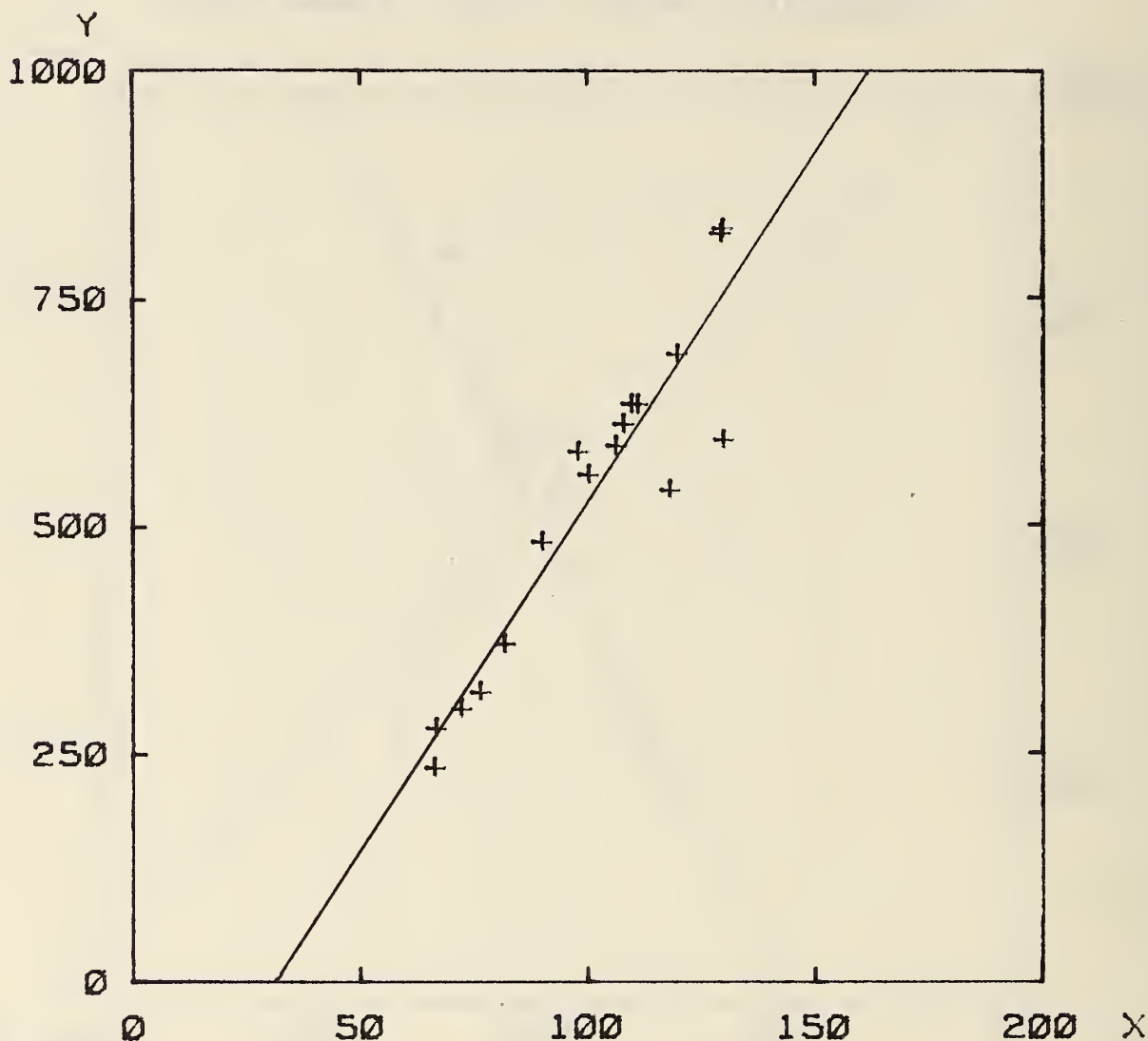
$$Y = -429.3296 + 0.9443 X$$

$$R\text{-SQUARED} = 0.8260$$

PROJECTIONS	Y	X
1985	1242.5560	1770.5000
2000	2207.1599	2792.0000

FIGURE F-15. - Scatter diagram of aluminum demand for electrical use and gross national product (GNP).

ALUMINUM DEMAND--ELECTRICAL
THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION
1967 = 100

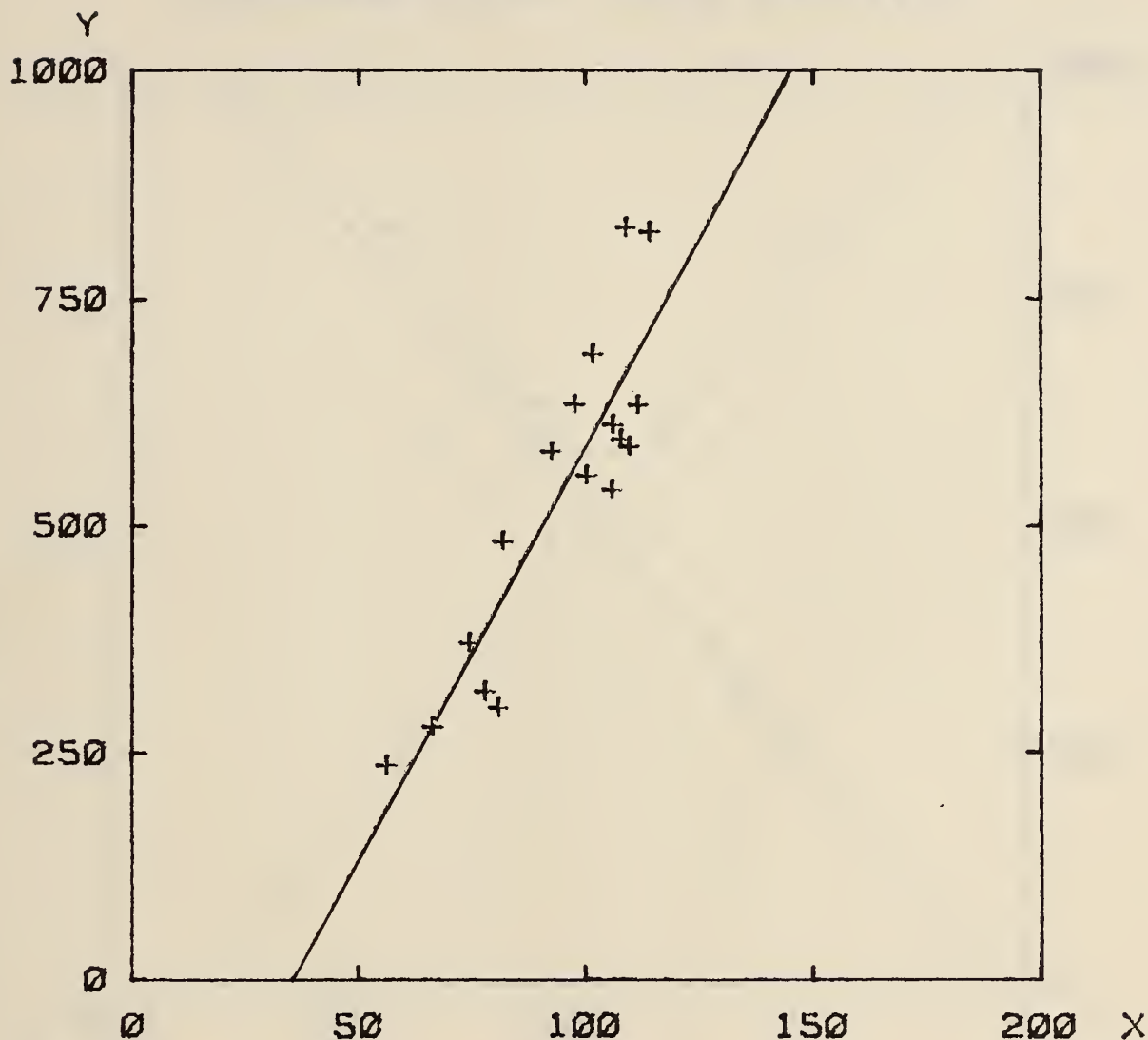
$$Y = -241.8019 + 7.6798 X$$

$$R\text{-SQUARED} = 0.8771$$

PROJECTIONS	Y	X
1985	1188.9499	186.3000
2000	2138.9447	310.0000

FIGURE F-16. - Scatter diagram of aluminum demand for electrical use and FRB index for total production.

ALUMINUM DEMAND--ELECTRICAL
THOUSAND SHORT TONS (1960-1976)



FRBI-COMMUNICATION EQUIPMENT
1967 = 100

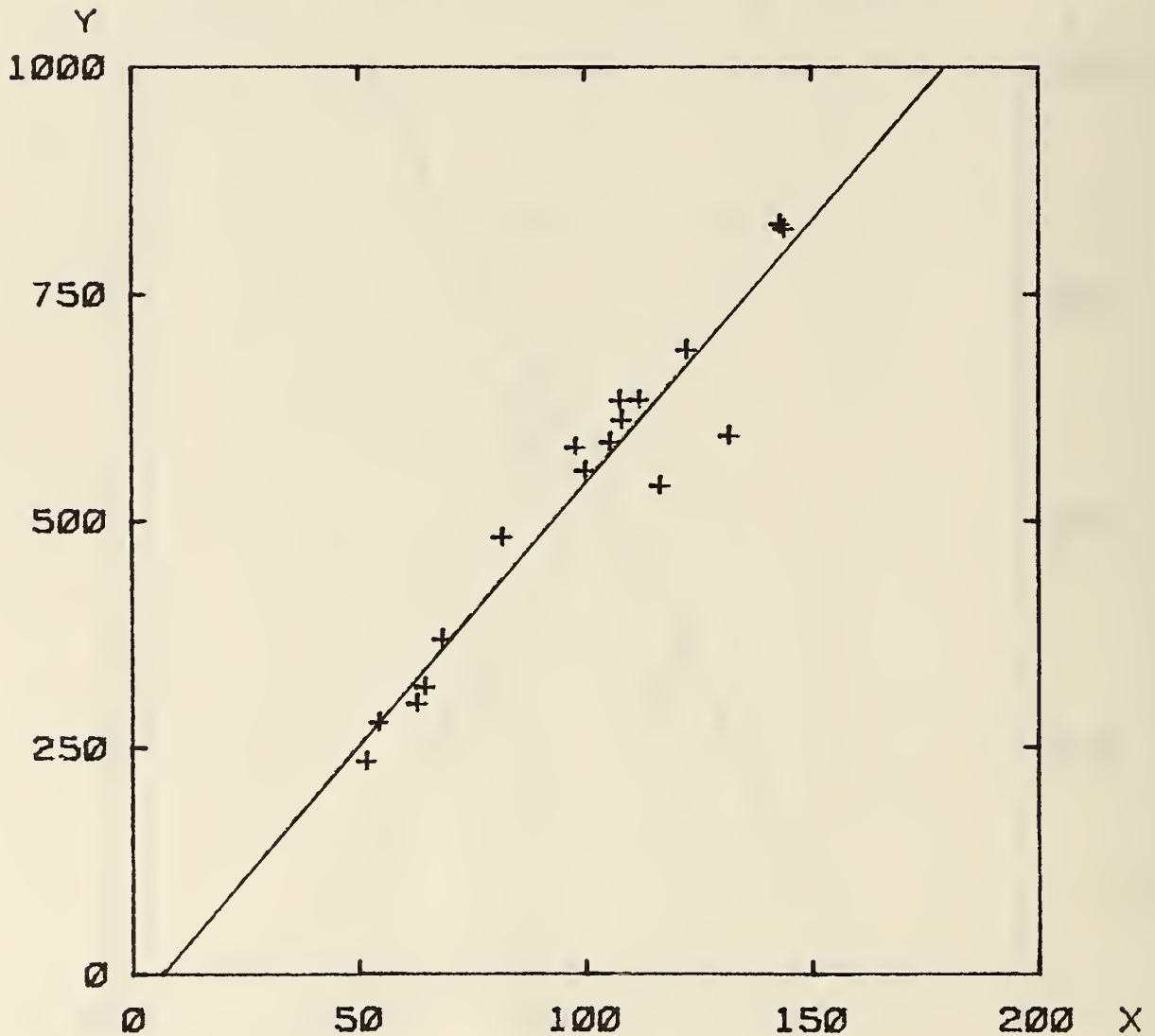
$$Y = - 323.7736 + 9.1414 X$$

$$R\text{-SQUARED} = 0.7993$$

PROJECTIONS	Y	X
1985	1062.0638	151.6000
2000	1842.7401	237.0000

FIGURE F-17. - Scatter diagram of aluminum demand for electrical use and FRB index for communication equipment.

ALUMINUM DEMAND--ELECTRICAL
THOUSAND SHORT TONS (1960-1976)



FRBI-ELECTRICAL MACHINERY
1967 = 100

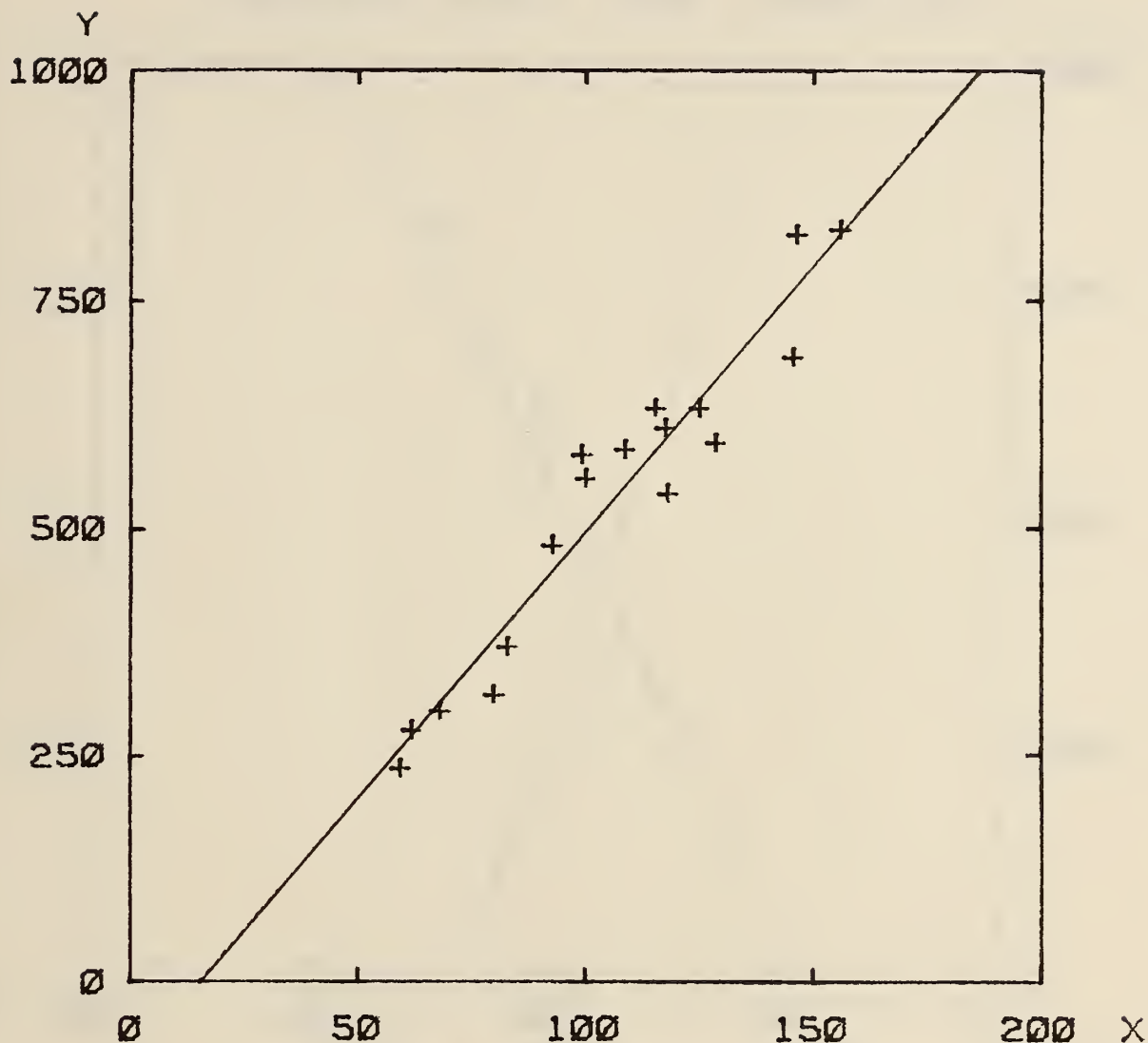
$$Y = -38.7270 + 5.7982 X$$

$$R\text{-SQUARED} = 0.9235$$

PROJECTIONS	Y	X
1985	987.5575	177.0000
2000	1410.8273	250.0000

FIGURE F-18. - Scatter diagram of aluminum demand for electrical use and FRB index for electrical machinery.

ALUMINUM DEMAND--ELECTRICAL
THOUSAND SHORT TONS (1960-1976)



FRBI-HOUSEHOLD APPLIANCES
1967 = 100

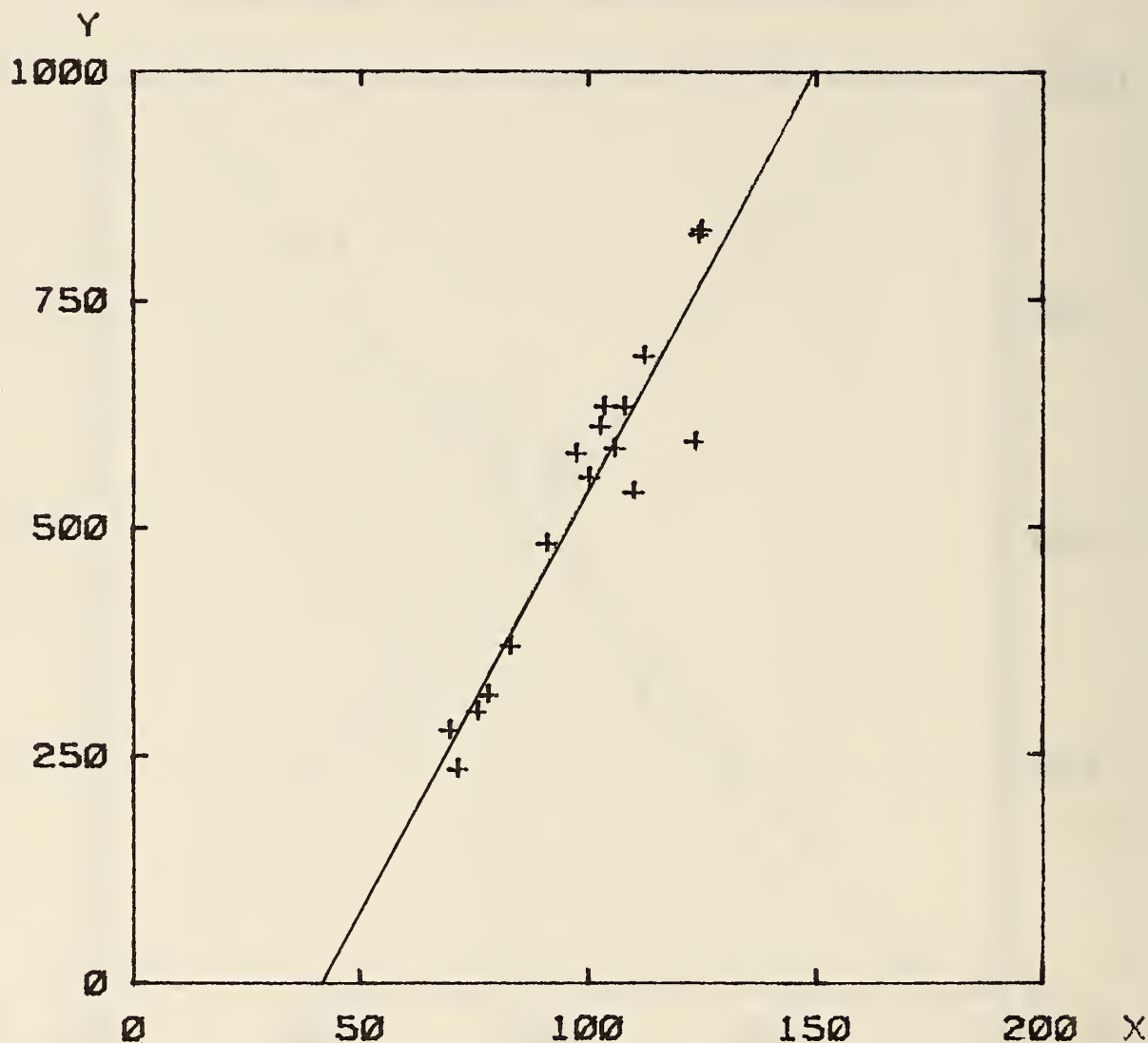
$$Y = -89.7623 + 5.8568 X$$

$$R\text{-SQUARED} = 0.9257$$

PROJECTIONS	Y	X
1985	1539.0213	278.1000
2000	2779.4973	489.9000

FIGURE F-19. - Scatter diagram of aluminum demand for electrical use and FRB index for household appliances.

ALUMINUM DEMAND--ELECTRICAL
THOUSAND SHORT TONS (1960-1976)



FRBI-FABRICATED METAL PRODUCTS
1967 = 100

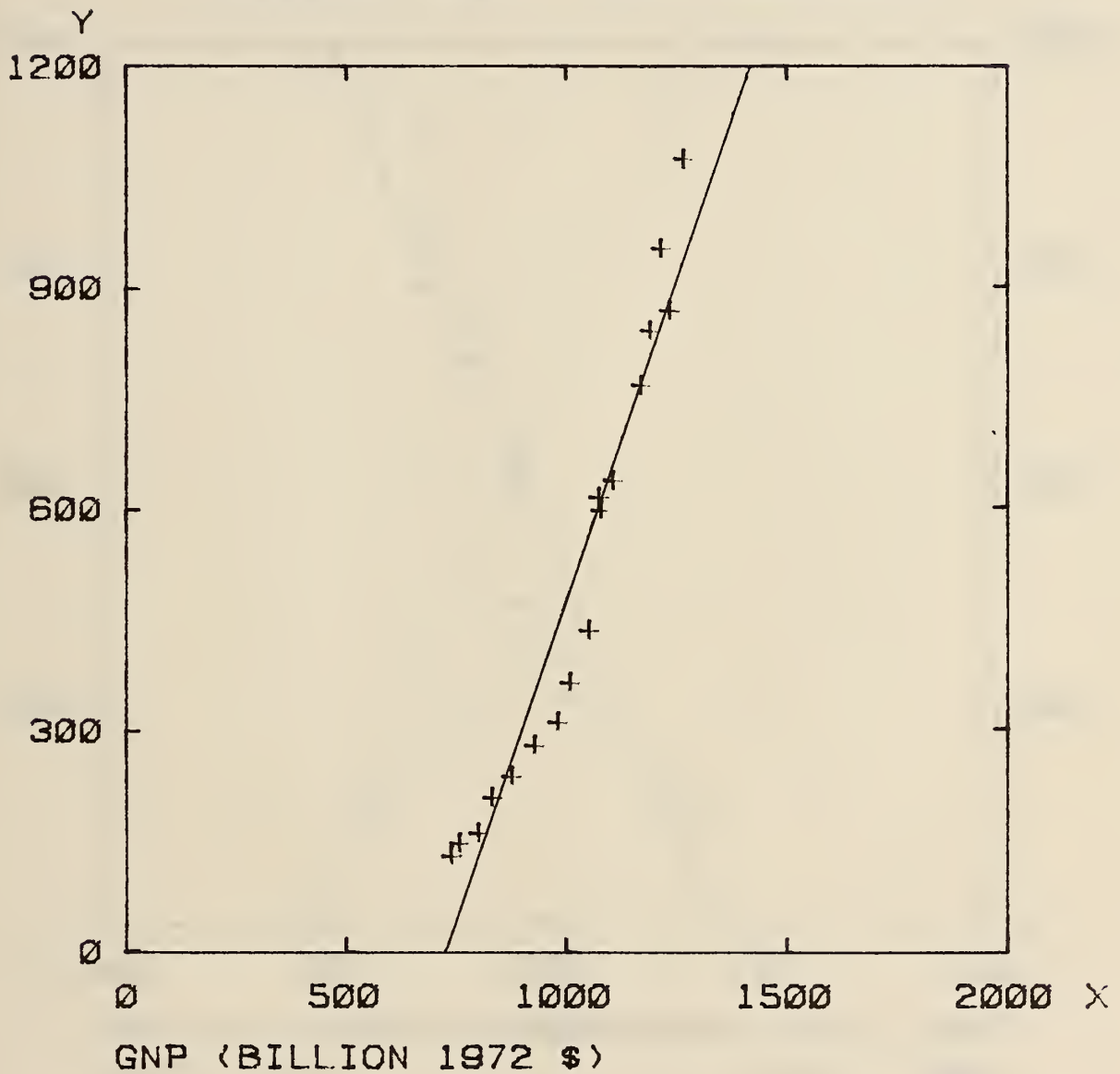
$$Y = - 384.0102 + 9.2771 X$$

$$R\text{-SQUARED} = 0.8899$$

PROJECTIONS	Y	X
1985	1255.2668	176.7000
2000	2028.0556	260.0000

FIGURE F-20. - Scatter diagram of aluminum demand for electrical use and FRB index for fabricated metal products.

ALUMINUM DEMAND-CANS AND CONTAINERS
THOUSAND SHORT TONS (1960-1976)



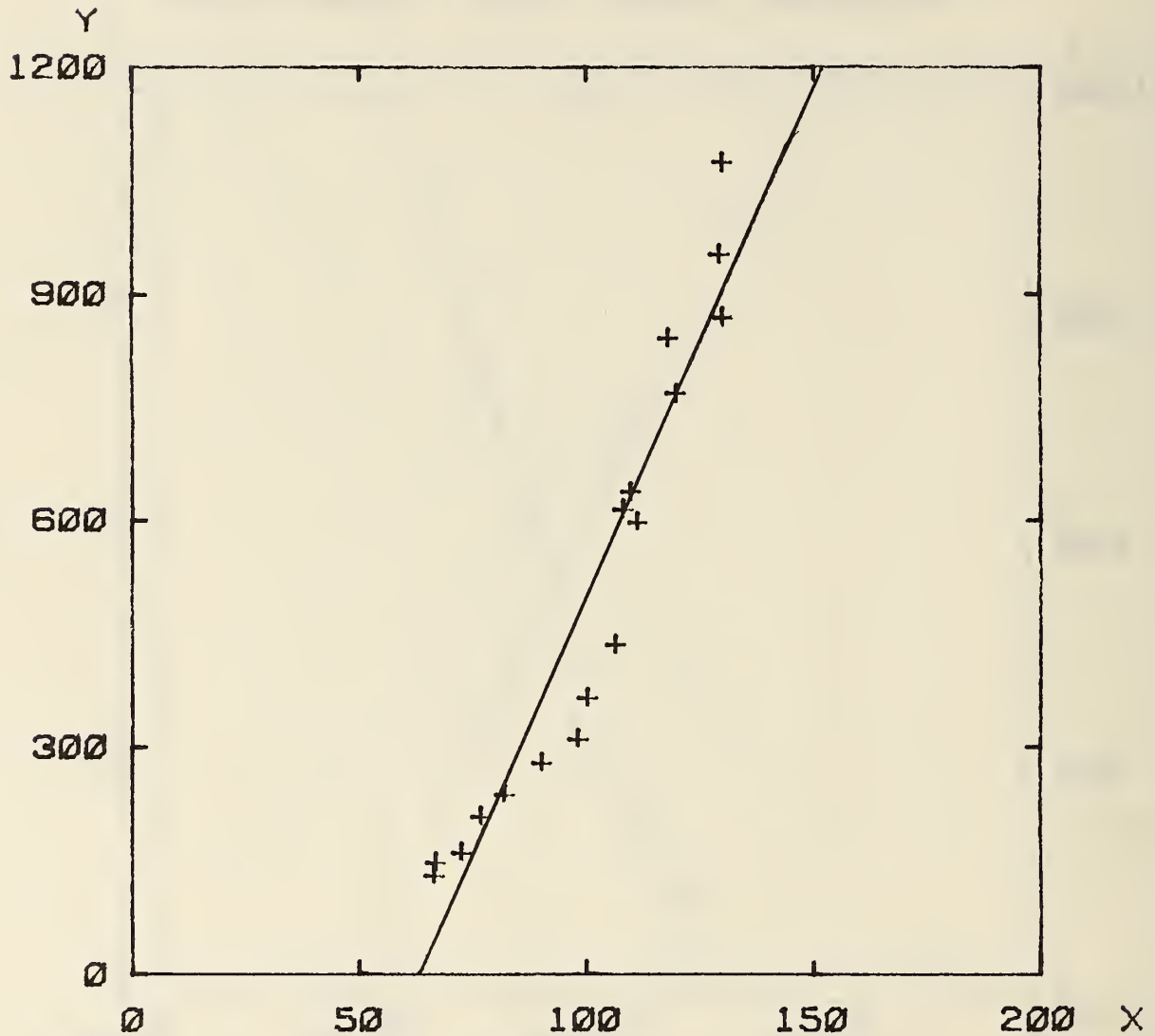
$$Y = -1251.4467 + 1.7276 X$$

$$R\text{-SQUARED} = 0.9271$$

PROJECTIONS	Y	X
1985	1807.3477	1770.5000
2000	3572.1366	2792.0000

FIGURE F-21. - Scatter diagram of aluminum demand for cans and containers use and gross national product (GNP).

ALUMINUM DEMAND--CANS AND CONTAINERS
THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION
1967 = 100

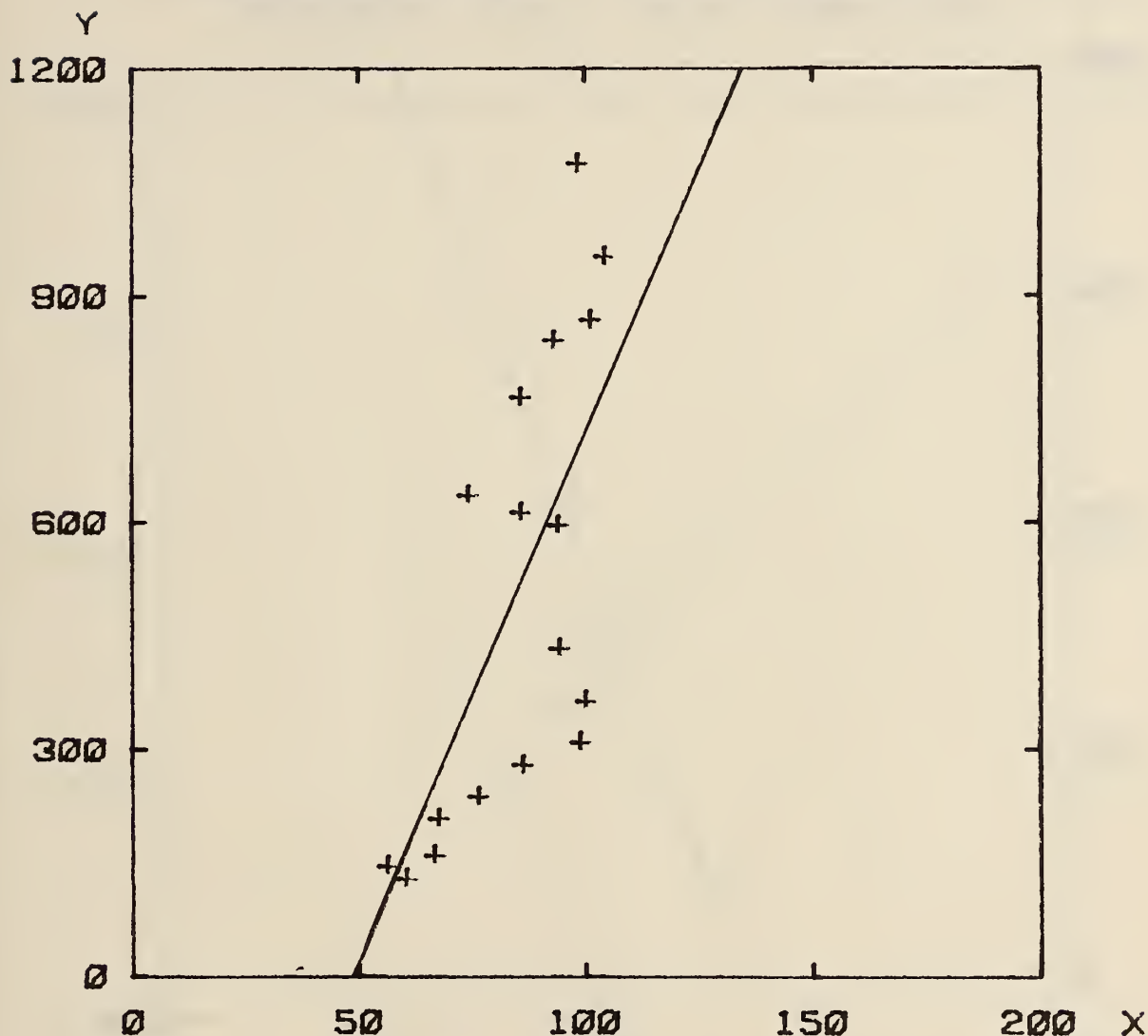
$$Y = -852.9654 + 13.5006 X$$

$$R\text{-SQUARED} = 0.9089$$

PROJECTIONS	Y	X
1985	1662.2002	186.3000
2000	3332.2270	310.0000

FIGURE F-22. - Scatter diagram of aluminum demand for cans and containers use and FRB index for total production.

ALUMINUM DEMAND--CANS AND CONTAINERS THOUSAND SHORT TONS (1960-1976)



FRBI-METALWORKING MACHINERY
1967 = 100

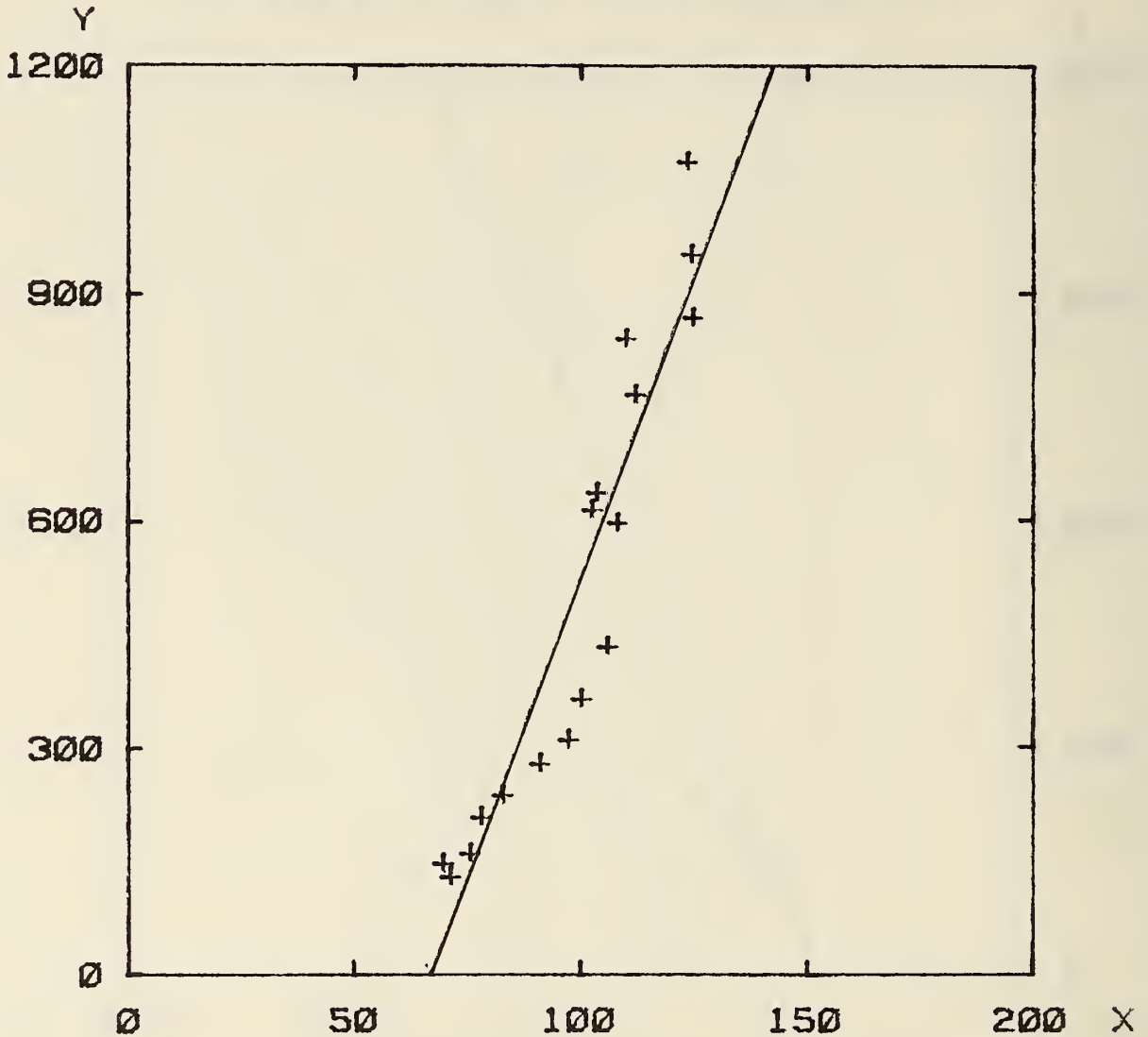
$$Y = - 679.7833 + 13.9800 X$$

$$R\text{-SQUARED} = 0.4706$$

PROJECTIONS	Y	X
1985	1512.2862	156.8000
2000	2488.0971	226.6000

FIGURE F-23. - Scatter diagram of aluminum demand for cans and containers use and FRB index for metalworking machinery.

ALUMINUM DEMAND--CANS AND CONTAINERS
THOUSAND SHORT TONS (1960-1976)



FRBI-FABRICATED METAL PRODUCTS
1967 = 100

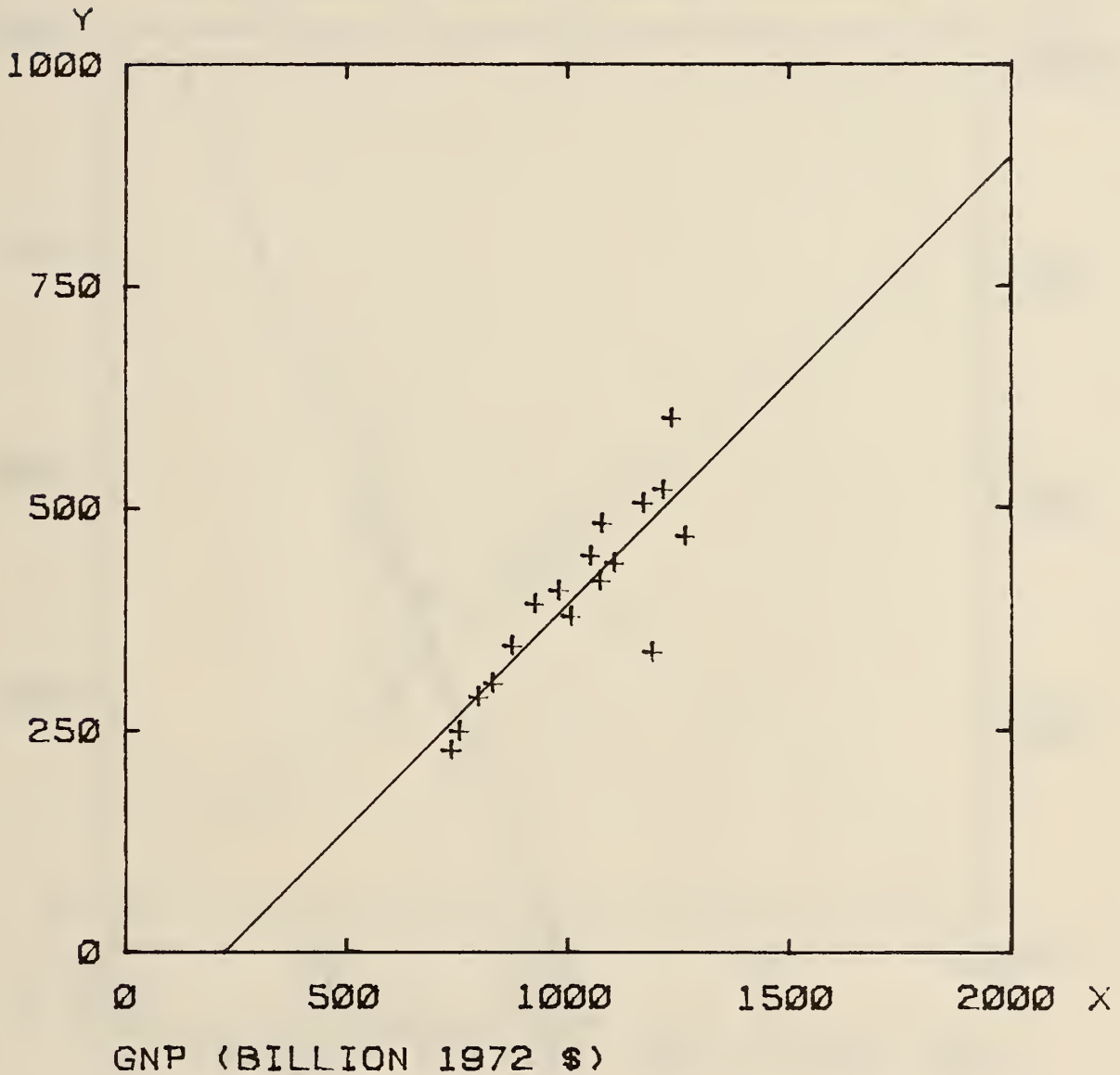
$$Y = -1063.7366 + 15.9112 X$$

$$R\text{-SQUARED} = 0.8778$$

PROJECTIONS	Y	X
1985	1747.7850	176.7000
2000	3073.1940	260.0000

FIGURE F-24. - Scatter diagram of aluminum demand for cans and containers use and FRB index for fabricated metal products.

ALUMINUM DEMAND-APPLIANCES AND EQUIPMENT THOUSAND SHORT TONS (1960-1976)

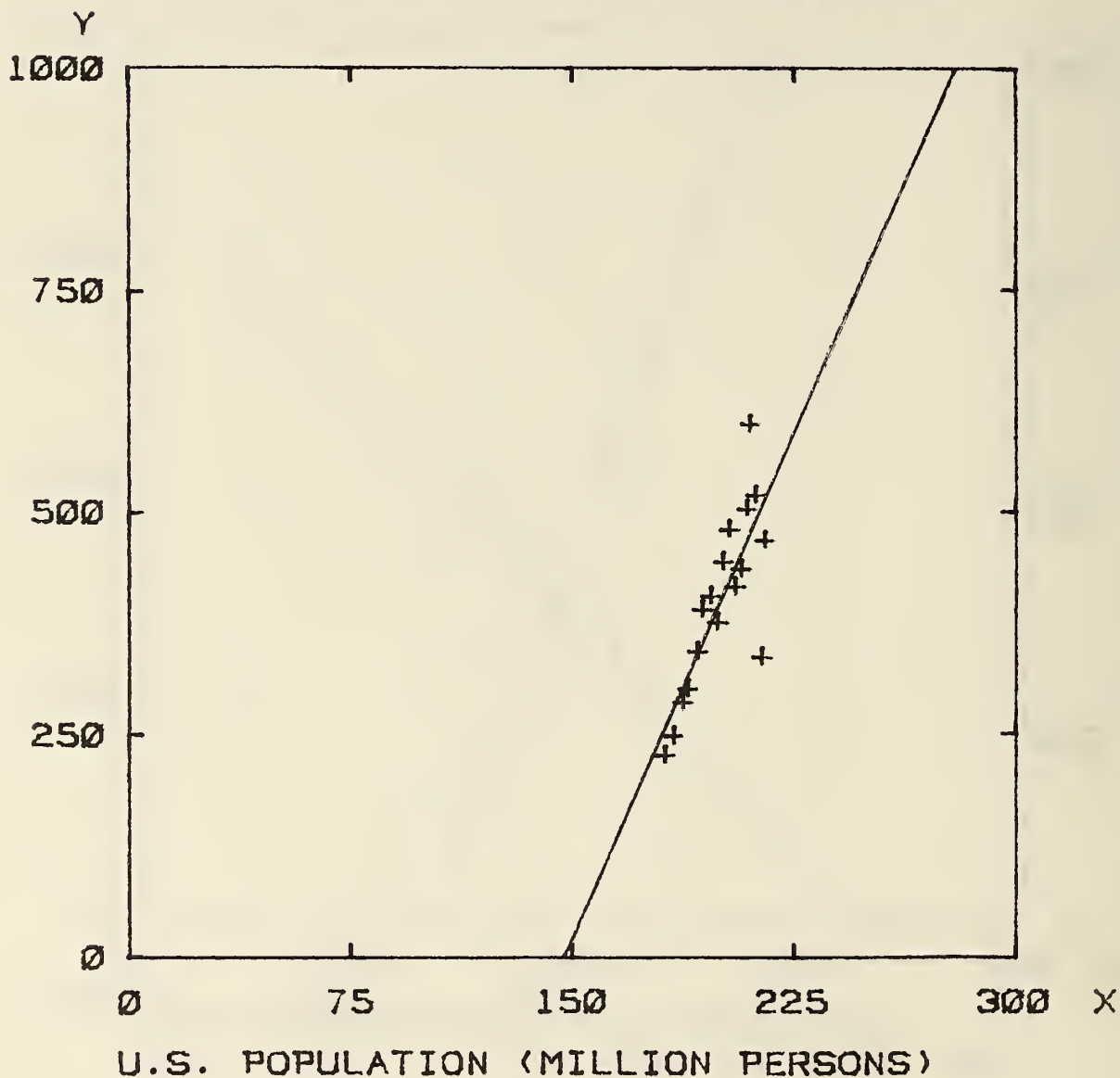


PROJECTIONS

	Y	X
1985	778.4576	1770.5000
2000	1293.4882	2792.0000

FIGURE F-25. - Scatter diagram of aluminum demand for appliances and equipment use and gross national product (GNP).

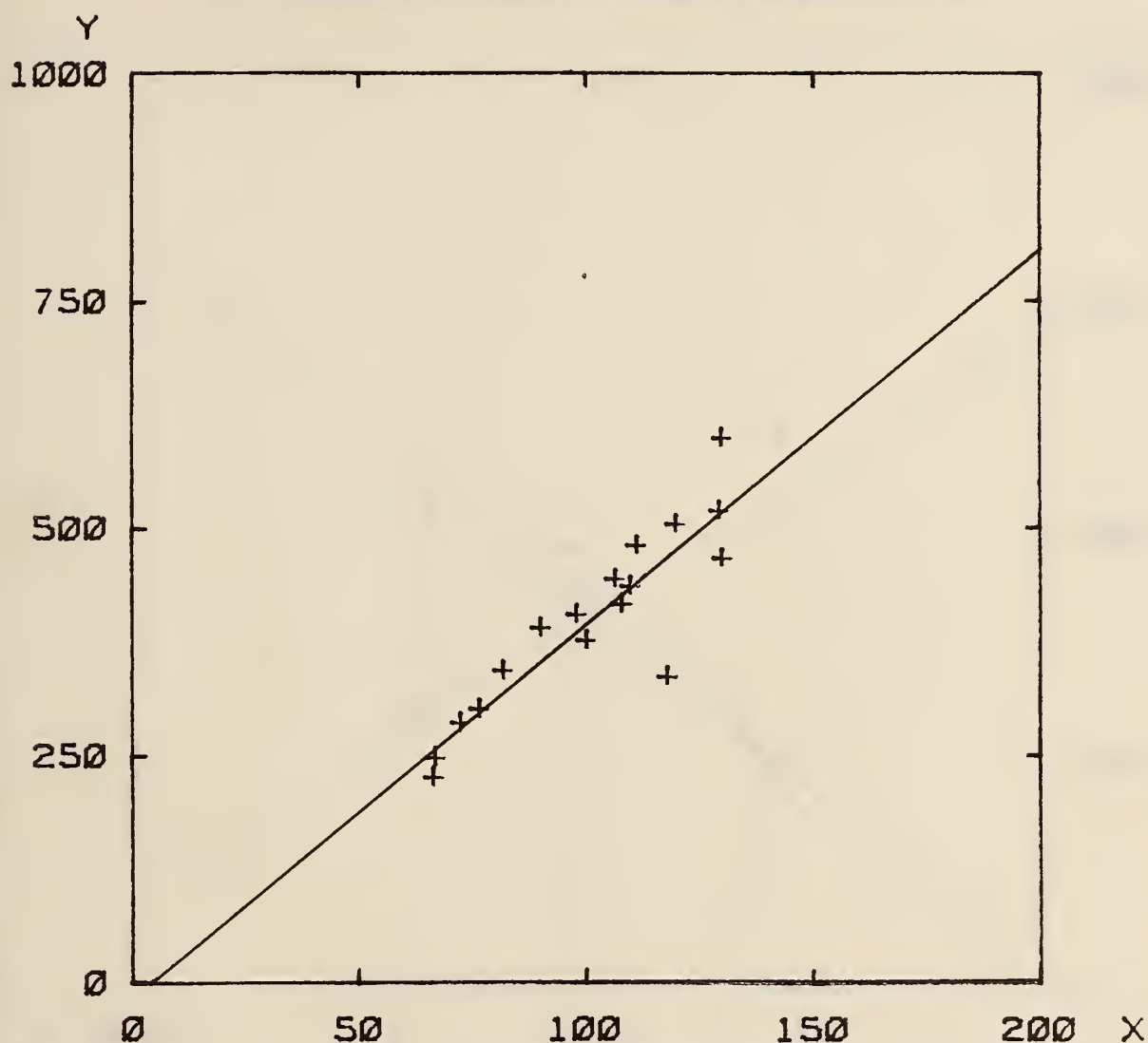
ALUMINUM DEMAND--APPLIANCES AND EQUIPMENT THOUSAND SHORT TONS (1960-1976)



PROJECTIONS	Y	X
1985	656.8400	234.1000
2000	871.3456	262.5000

FIGURE F-26. - Scatter diagram of aluminum demand for appliances and equipment use and U.S. population.

ALUMINUM DEMAND--APPLIANCES AND EQUIPMENT THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION

1967 = 100

$$Y = -17.0310 + 4.1297 X$$

$$R\text{-SQUARED} = 0.7974$$

PROJECTIONS

1985

752.3395

186.3000

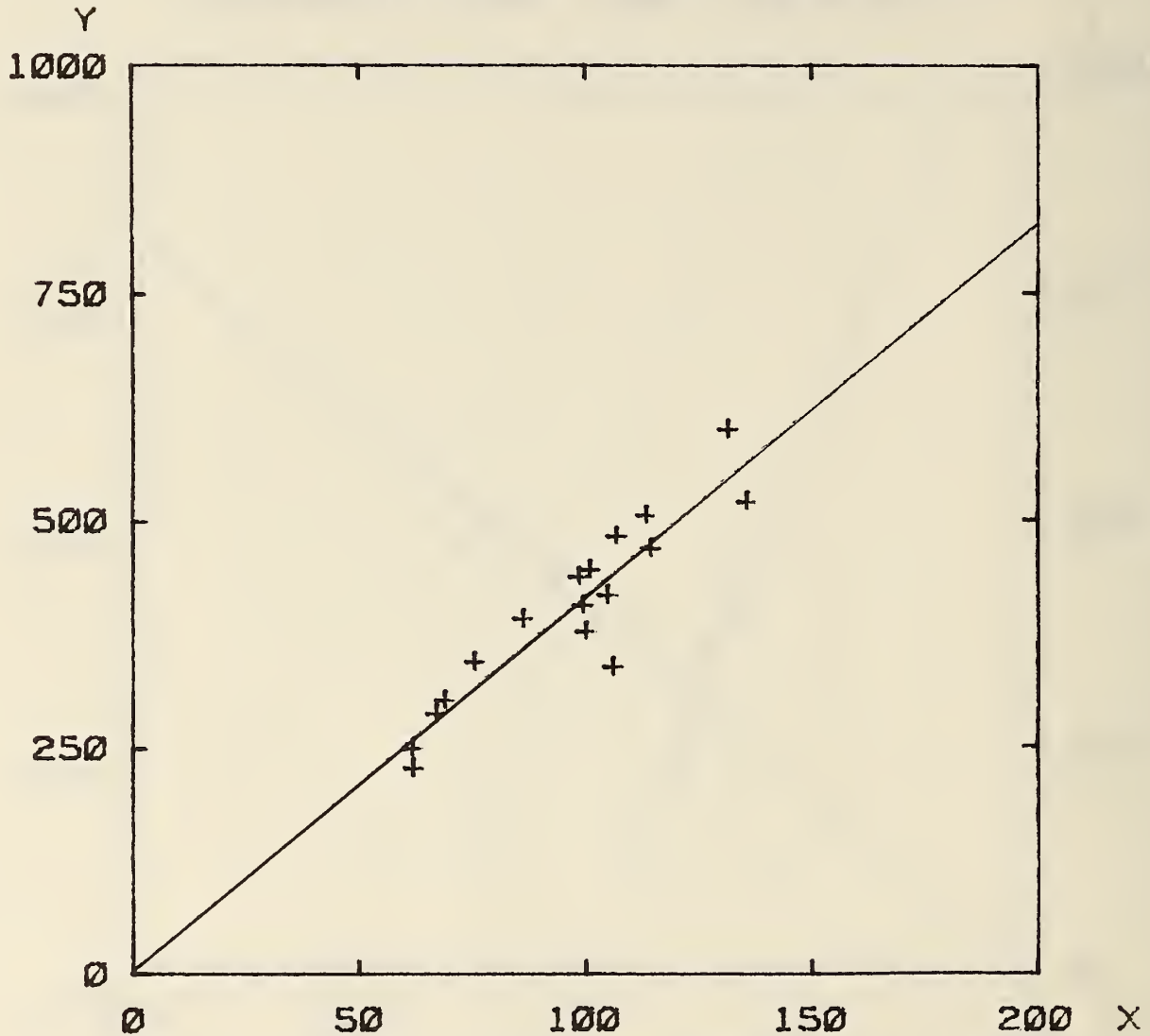
2000

1263.1883

310.0000

FIGURE F-27. - Scatter diagram of aluminum demand for appliances and equipment use and FRB index for total production.

ALUMINUM DEMAND--APPLIANCES AND EQUIPMENT THOUSAND SHORT TONS (1960-1976)



FRBI-MAJOR ELECTRICAL EQUIPMENT AND PARTS
1967 = 100

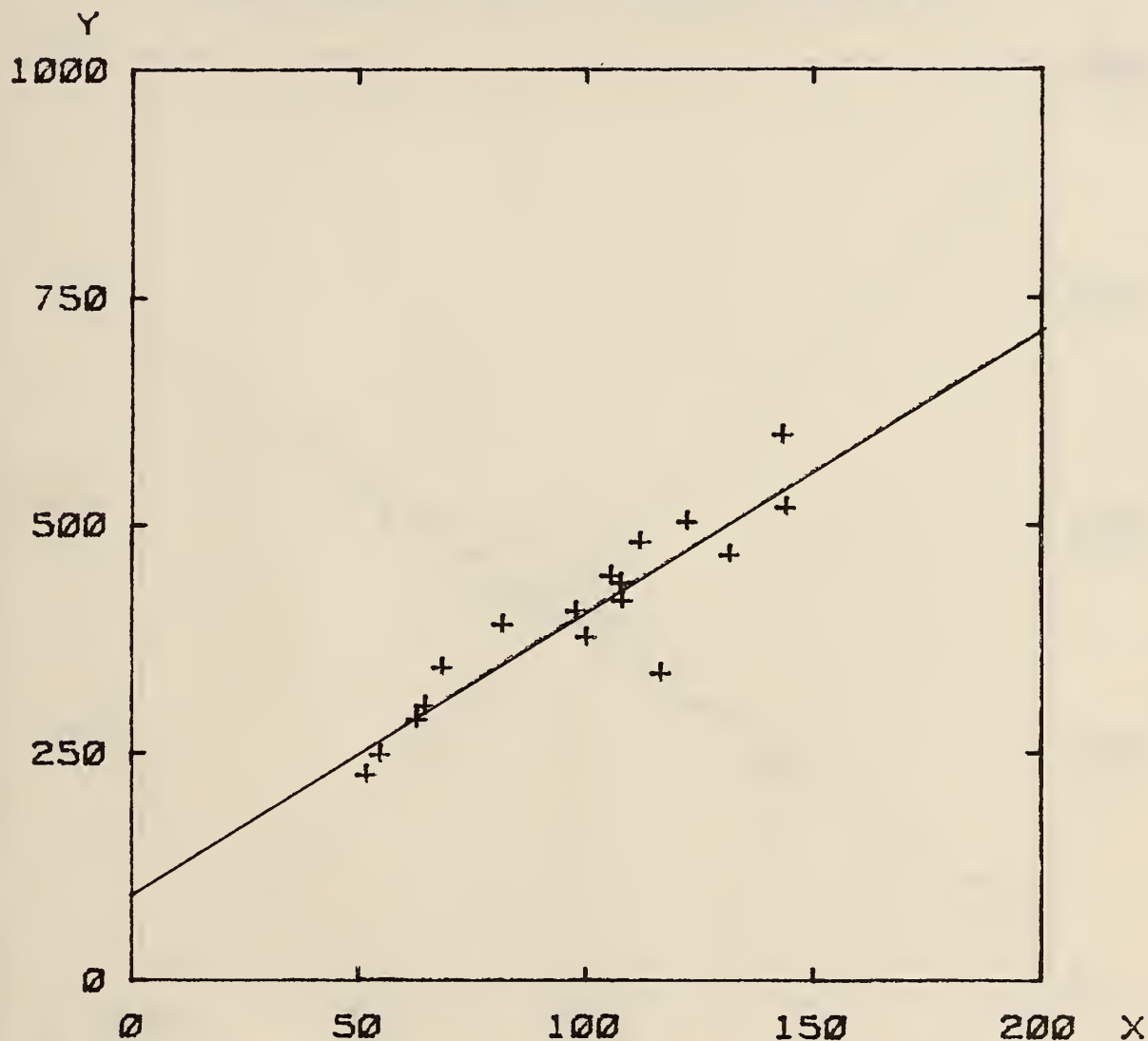
$$Y = 3.7347 + 4.1120 X$$

$$R\text{-SQUARED} = 0.8491$$

PROJECTIONS	Y	X
1985	716.3598	173.3000
2000	945.4031	229.0000

FIGURE F-28. - Scatter diagram of aluminum demand for appliances and equipment use and FRB index for major electrical equipment and parts.

ALUMINUM DEMAND--APPLIANCES AND EQUIPMENT THOUSAND SHORT TONS (1960-1976)



FRBI-ELECTRICAL MACHINERY
1967 = 100

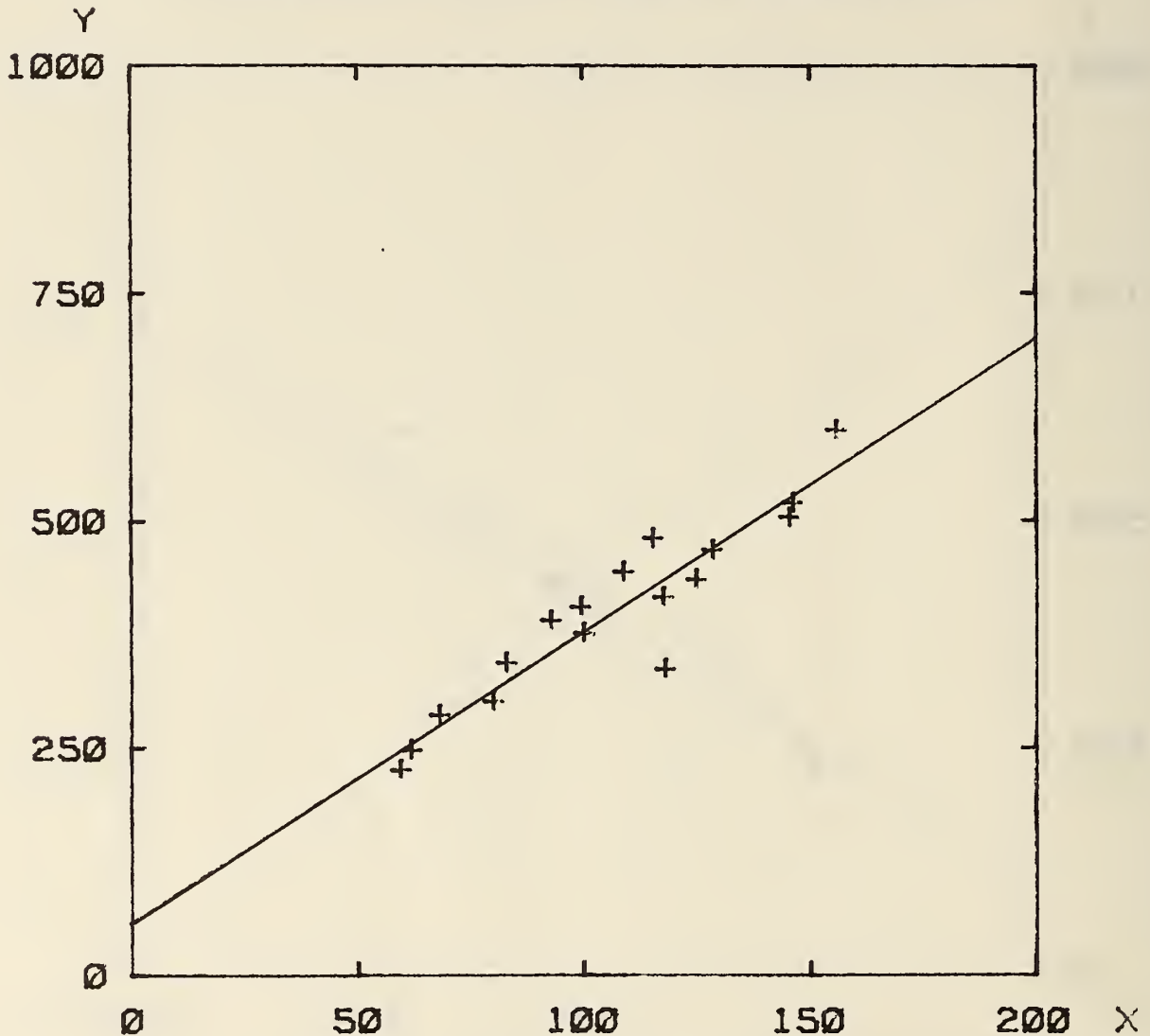
$$Y = 93.9233 + 3.1001 X$$

$$R\text{-SQUARED} = 0.8300$$

PROJECTIONS	Y	X
1985	642.6416	177.0000
2000	868.9491	250.0000

FIGURE F-29. - Scatter diagram of aluminum demand for appliances and equipment use and FRB index for electrical machinery.

ALUMINUM DEMAND--APPLIANCES AND EQUIPMENT THOUSAND SHORT TONS (1960-1976)



FRBI-HOUSEHOLD APPLIANCES
1967 = 100

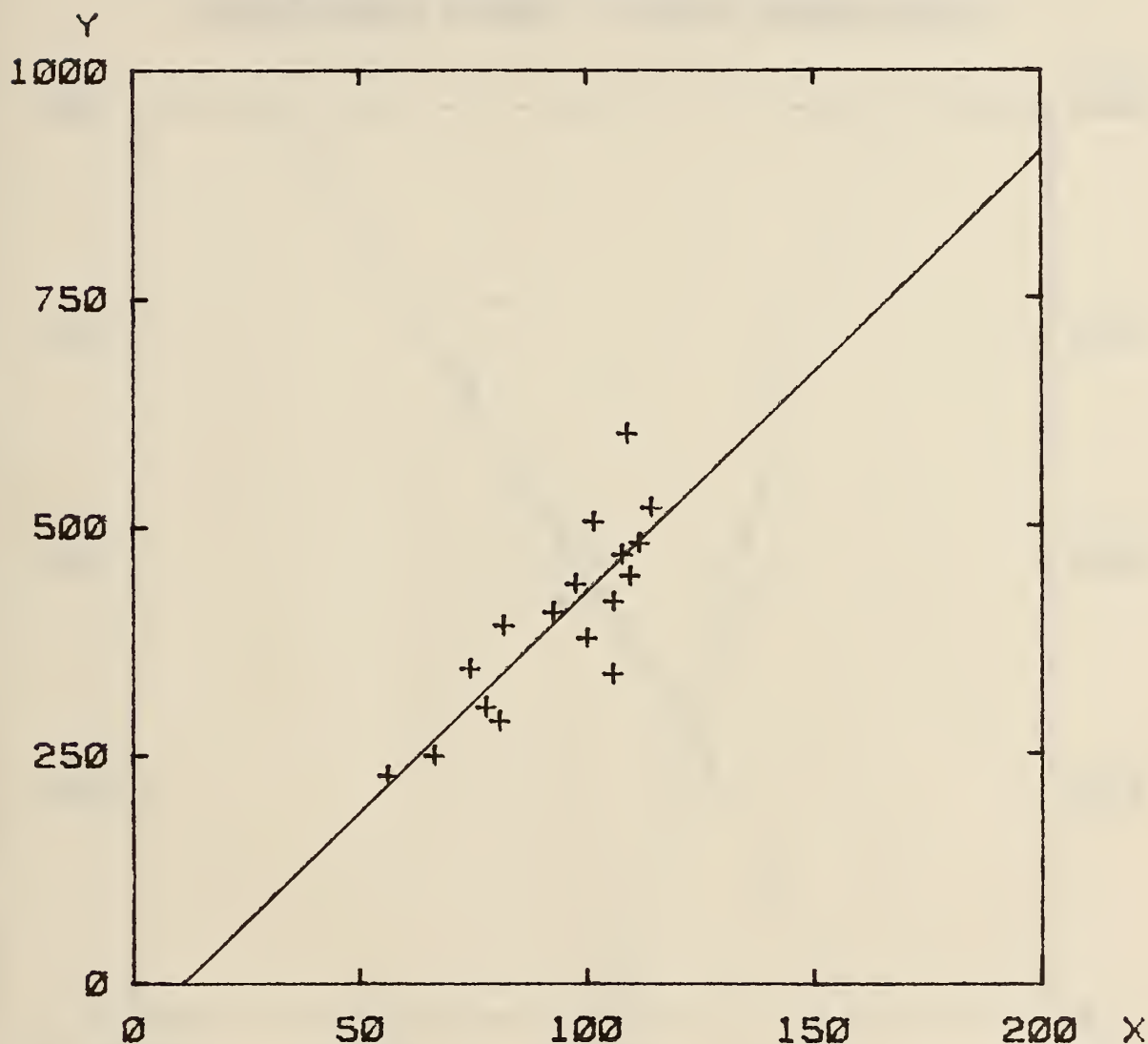
$$Y = 58.0345 + 3.2125 X$$

$$R\text{-SQUARED} = 0.8757$$

PROJECTIONS	Y	X
1985	951.4348	278.1000
2000	1631.8454	489.9000

FIGURE F-30. - Scatter diagram of aluminum demand for appliances and equipment use and FRB index for household appliances.

ALUMINUM DEMAND--APPLIANCES AND EQUIPMENT THOUSAND SHORT TONS (1960-1976)



FRBI-COMMUNICATION EQUIPMENT
1967 = 100

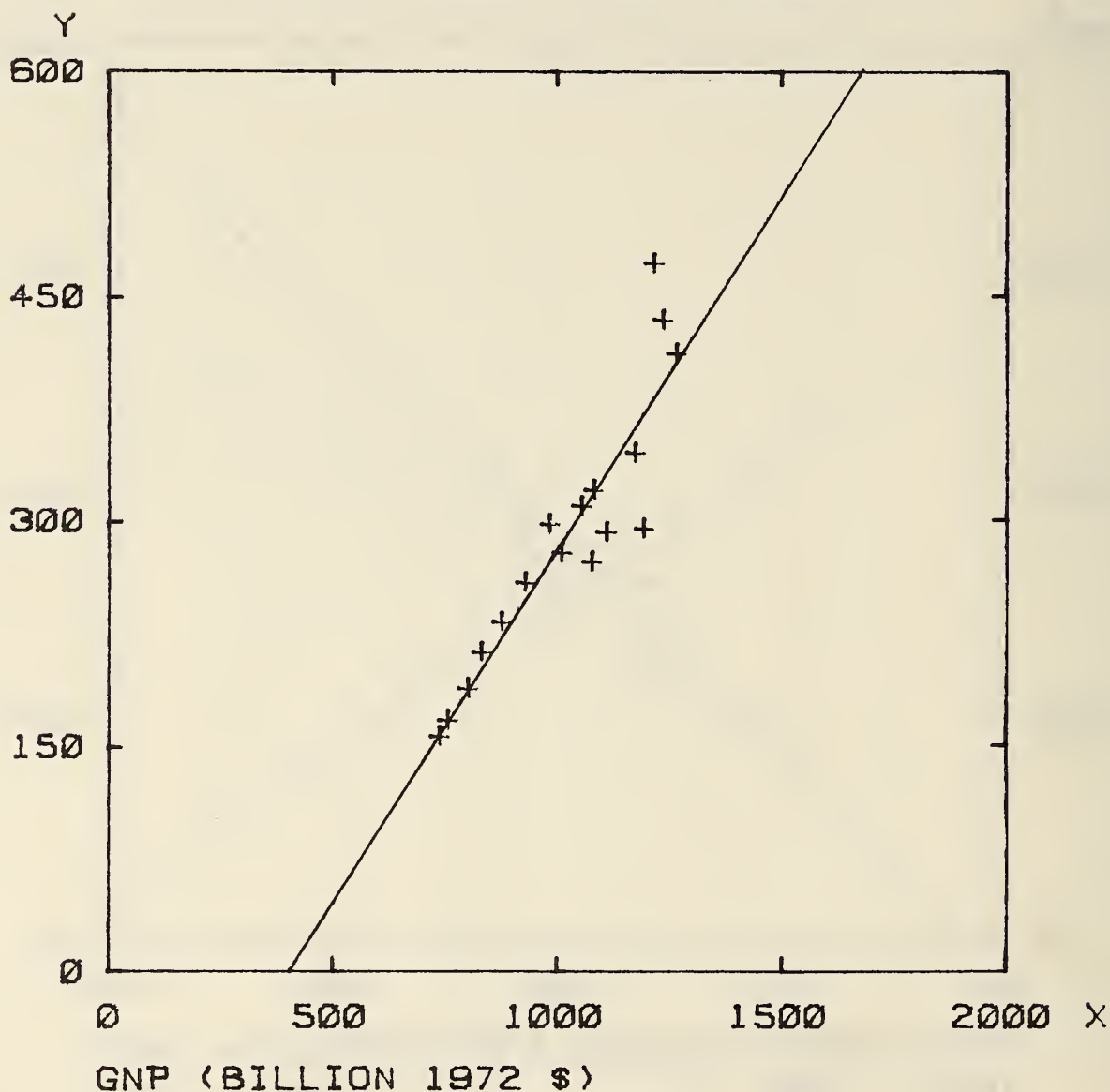
$$Y = - 52.8626 + 4.8275 X$$

$$R\text{-SQUARED} = 0.7009$$

PROJECTIONS	Y	X
1985	678.9938	151.6000
2000	1091.2665	237.0000

FIGURE F-31. - Scatter diagram of aluminum demand for appliances and equipment use and FRB index for communication equipment.

ALUMINUM DEMAND--MACHINERY
THOUSAND SHORT TONS (1960-1976)



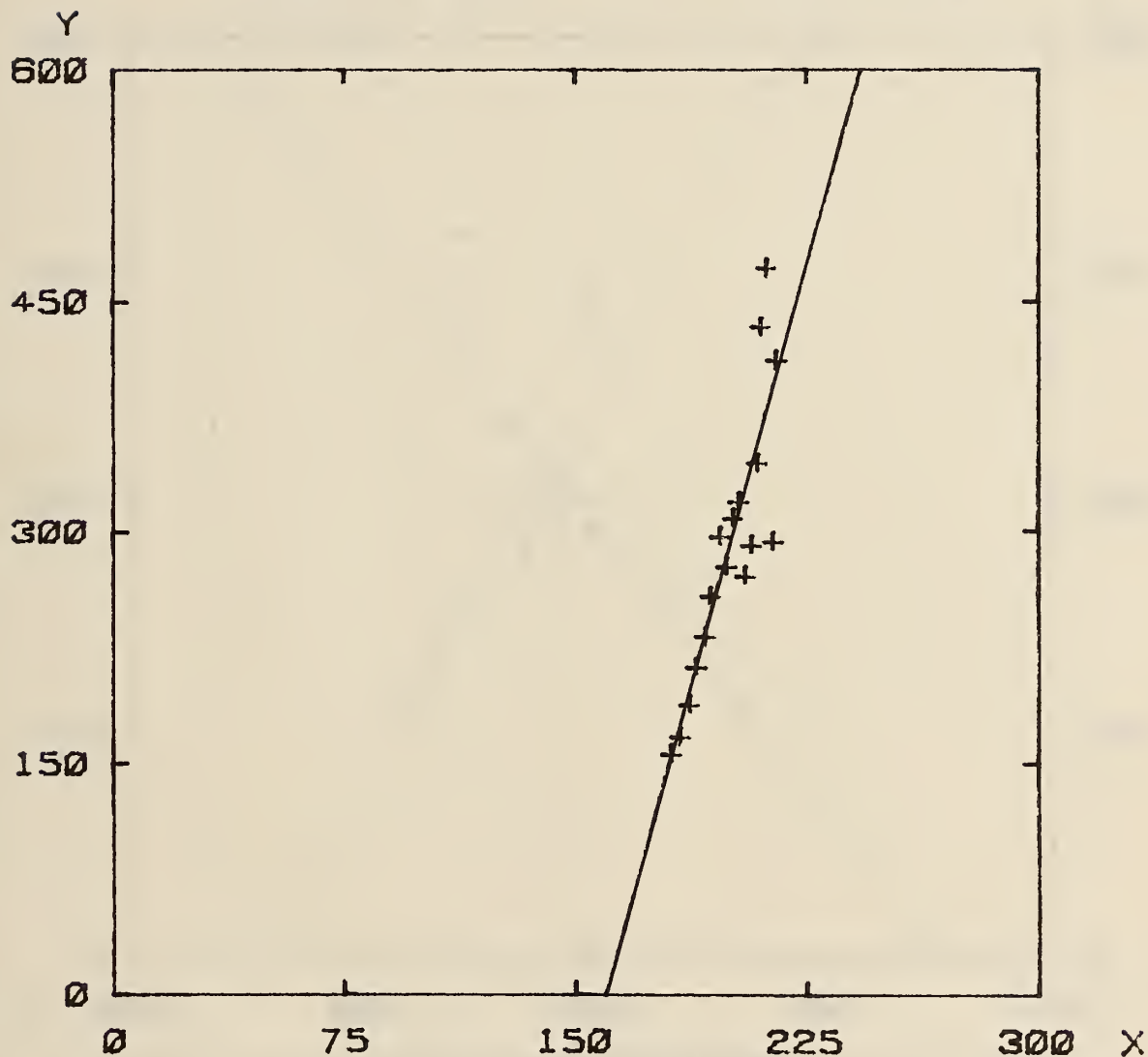
$$Y = -190.7349 + 0.4728 X$$

$$R\text{-SQUARED} = 0.8373$$

PROJECTIONS	Y	X
1985	646.4556	1770.5000
2000	1129.4774	2792.0000

FIGURE F-32. - Scatter diagram of aluminum demand for machinery use and gross national product (GNP).

ALUMINUM DEMAND--MACHINERY
THOUSAND SHORT TONS (1960-1976)



U.S. POPULATION (MILLION PERSONS)

$$Y = - 1157.3510 + 7.2410 X$$

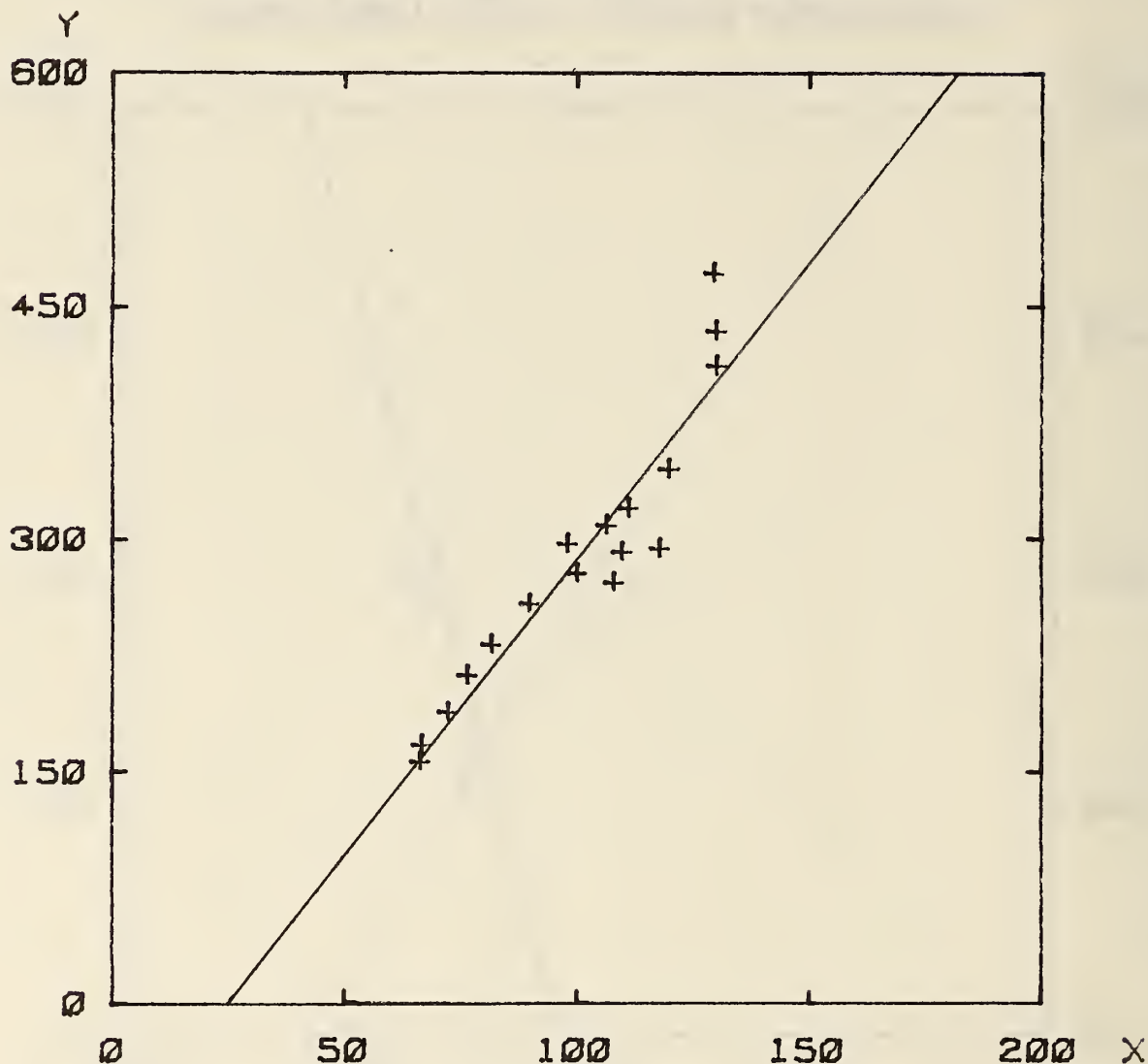
$$R\text{-SQUARED} = 0.7606$$

PROJECTIONS

	Y	X
1985	537.7728	234.1000
2000	743.4179	262.5000

FIGURE F-33. - Scatter diagram of aluminum demand for machinery use and U.S. population.

ALUMINUM DEMAND--MACHINERY
THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION
1967 = 100

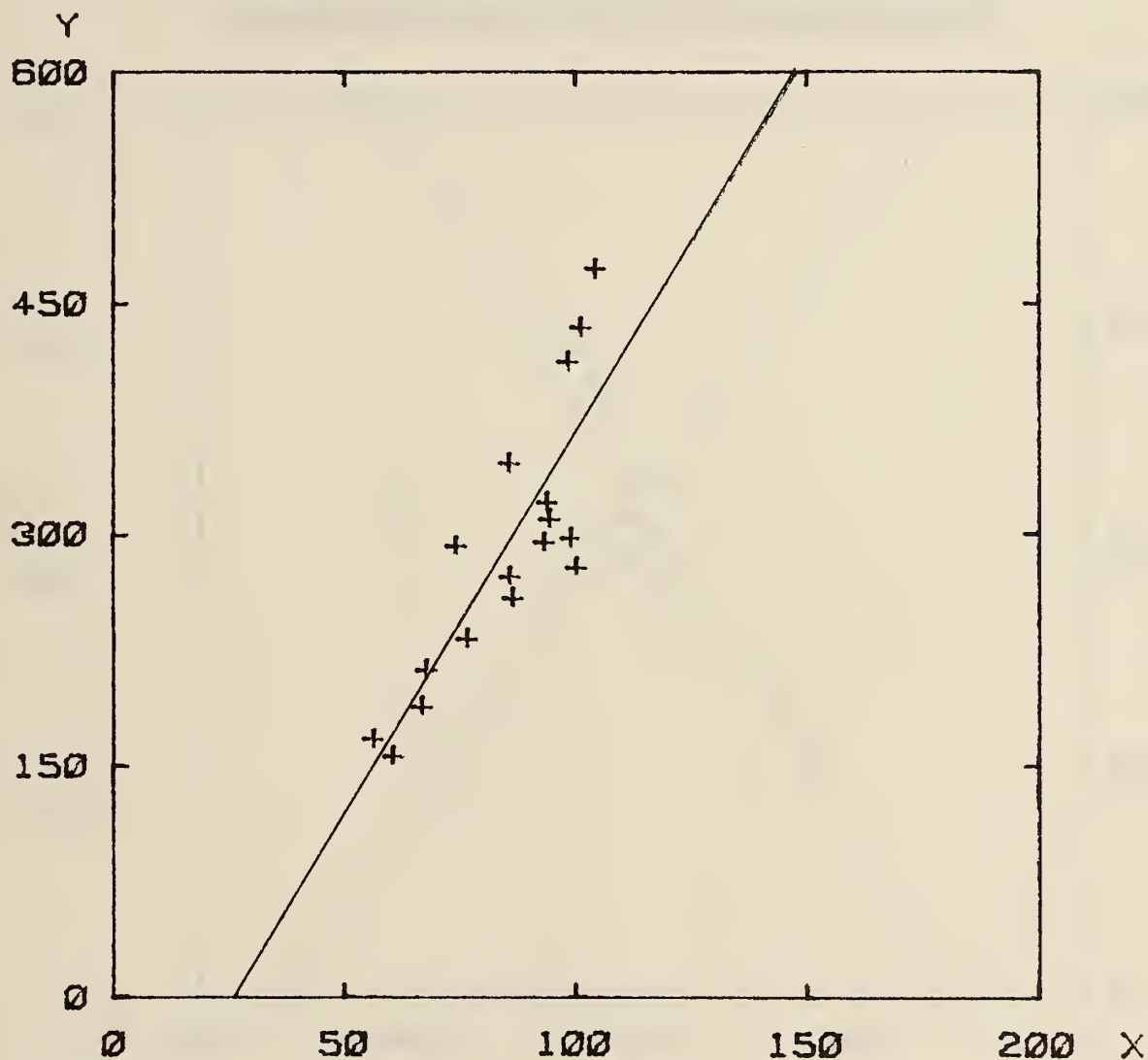
$$Y = -95.8003 + 3.8354 X$$

$$R\text{-SQUARED} = 0.8844$$

PROJECTIONS	Y	X
1985	618.7365	186.3000
2000	1093.1768	310.0000

FIGURE F-34. - Scatter diagram of aluminum demand for machinery use and FRB index for total production.

ALUMINUM DEMAND--MACHINERY
THOUSAND SHORT TONS (1960-1976)



FRBI-METALWORKING MACHINERY
1967 = 100

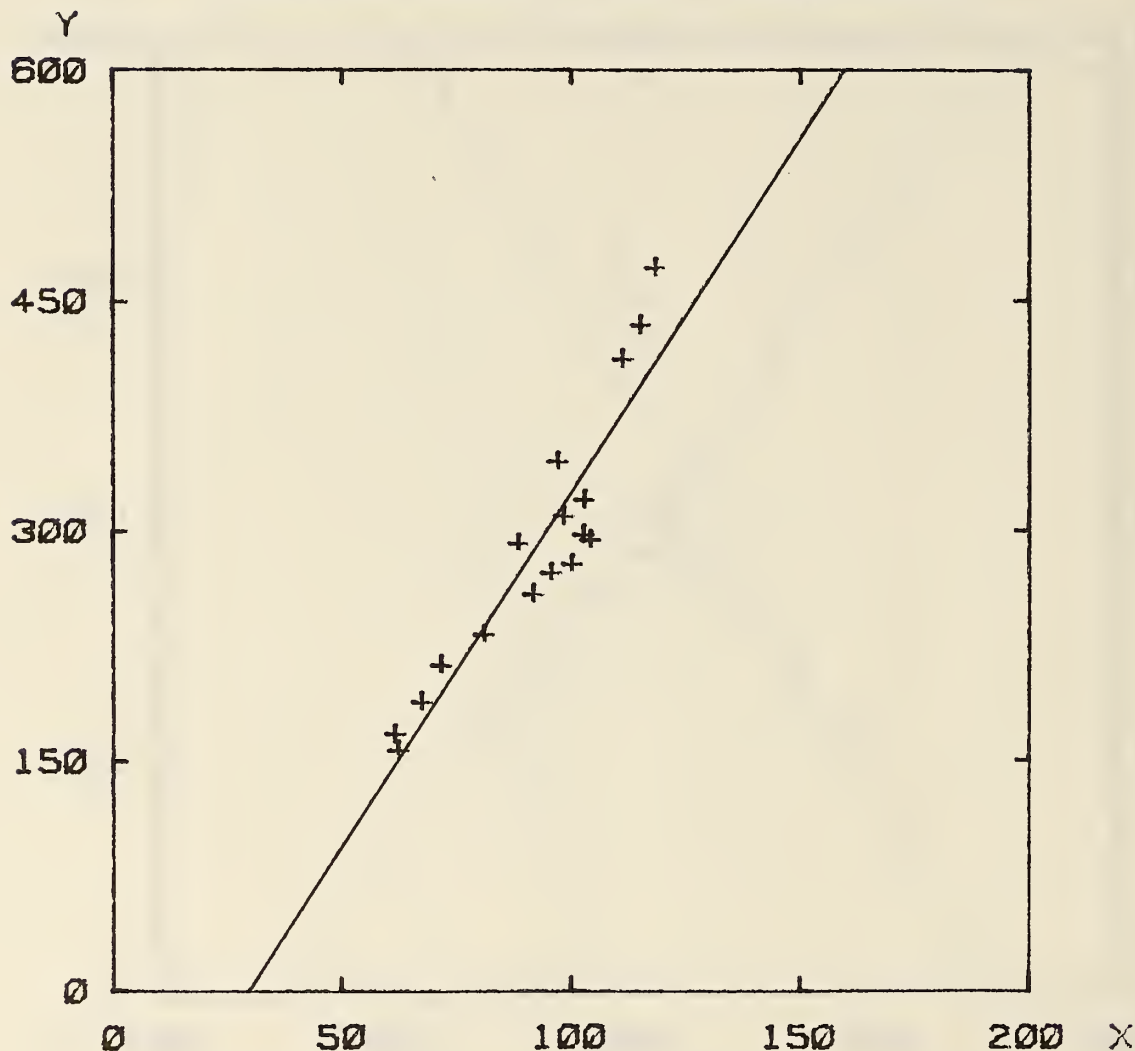
$$Y = -129.1799 + 4.9445 X$$

$$R\text{-SQUARED} = 0.7098$$

PROJECTIONS	Y	X
1985	646.1323	156.8000
2000	891.2649	226.6000

FIGURE F-35. - Scatter diagram of aluminum demand for machinery use and FRB index for metalworking machinery.

ALUMINUM DEMAND--MACHINERY
THOUSAND SHORT TONS (1960-1976)



FRBI-SPECIAL AND GENERAL INDUSTRIAL EQUIPMENT
1967 = 100

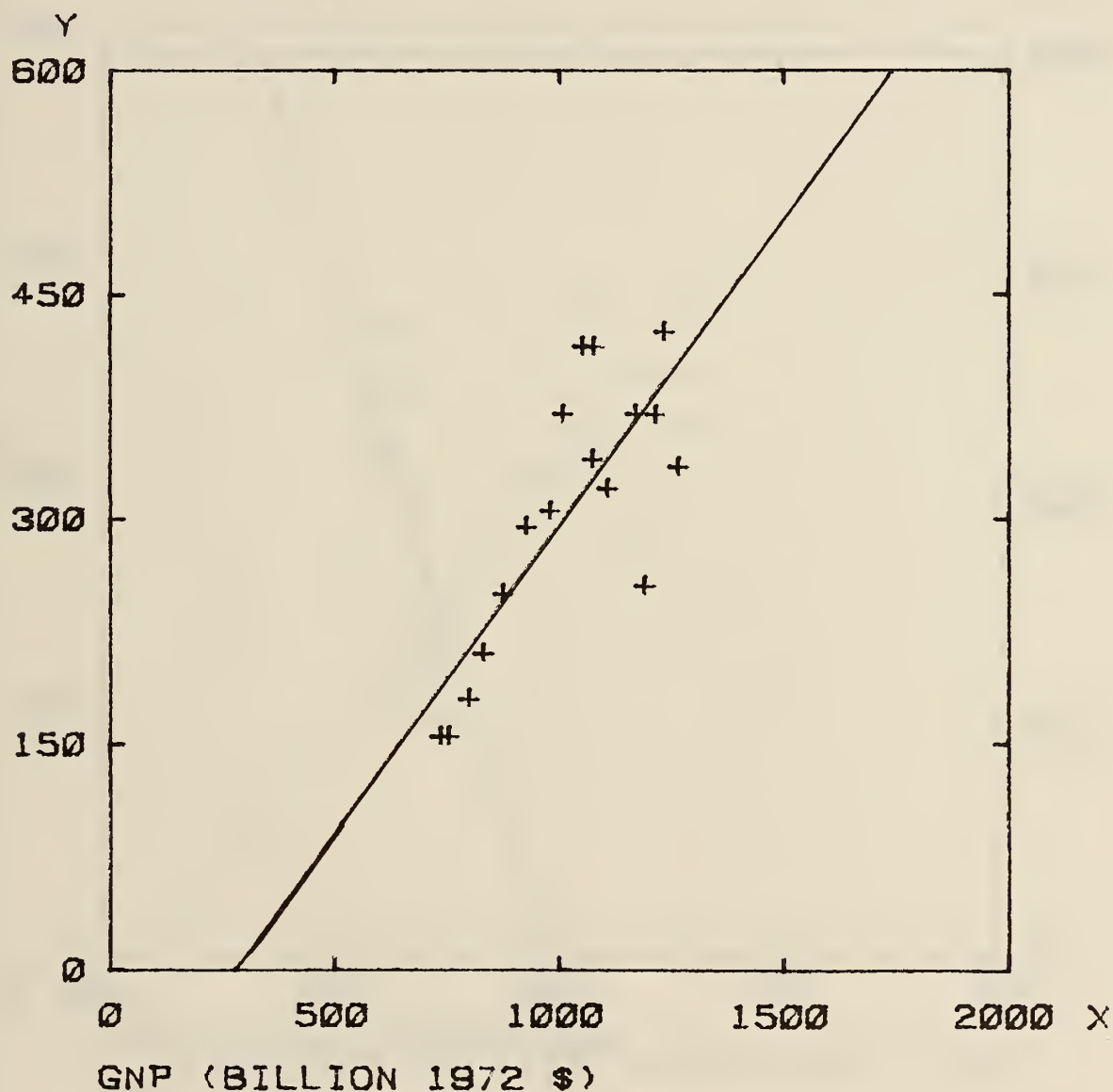
$$Y = -137.3615 + 4.6364 X$$

$$R\text{-SQUARED} = 0.8601$$

PROJECTIONS	Y	X
1985	769.9851	195.7000
2000	1318.0095	313.9000

FIGURE F-36. - Scatter diagram of aluminum demand for machinery use and FRB index for special and general industrial equipment.

ALUMINUM DEMAND--DISSIPATIVE
THOUSAND SHORT TONS (1960-1976)



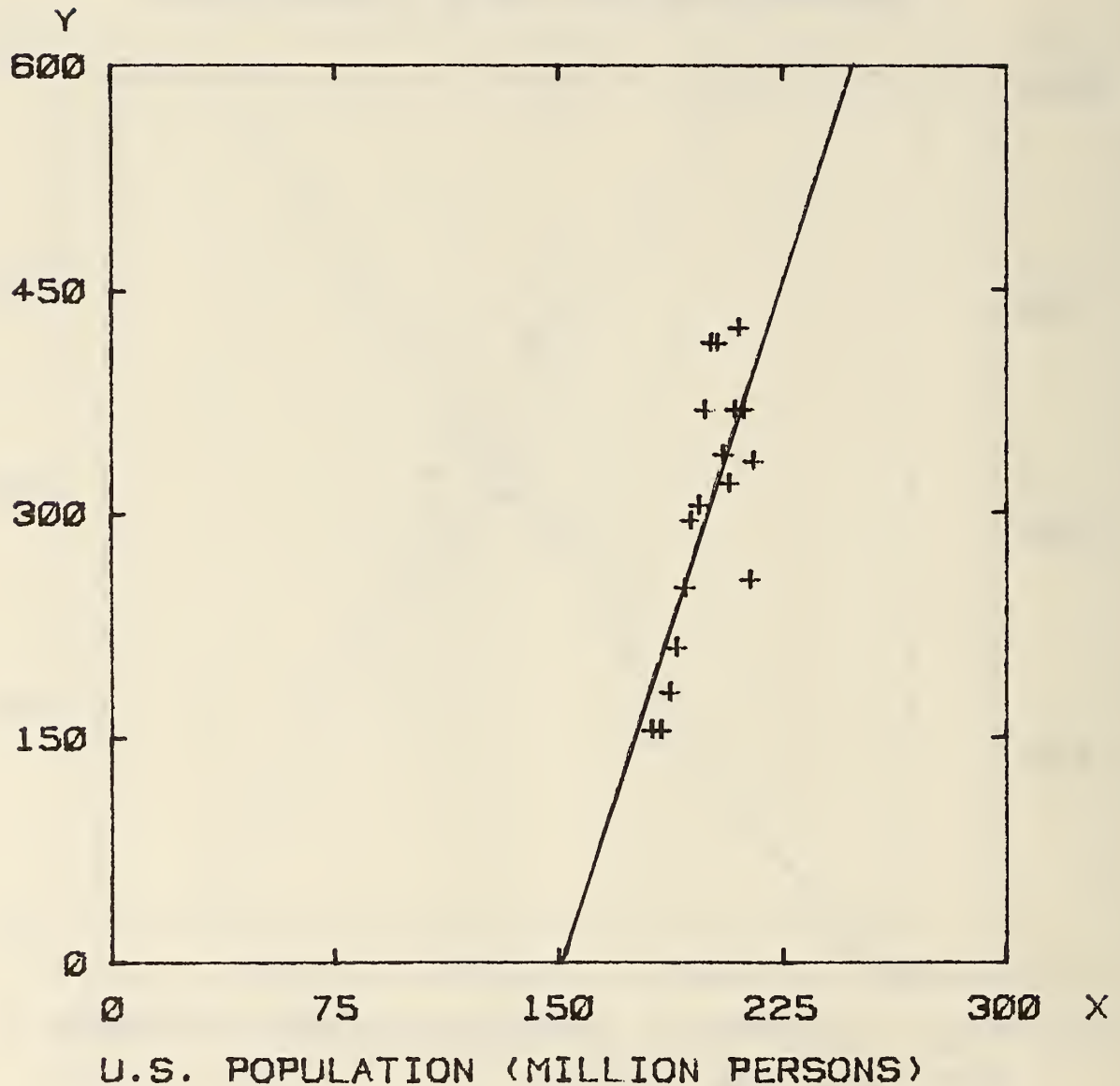
$$Y = -114.0878 + 0.4106 X$$

$$R\text{-SQUARED} = 0.6249$$

PROJECTIONS	Y	X
1985	612.9849	1770.5000
2000	1032.4737	2792.0000

FIGURE F-37. - Scatter diagram of aluminum demand for dissipative use and gross national product (GNP).

ALUMINUM DEMAND--DISSIPATIVE
THOUSAND SHORT TONS (1960-1976)



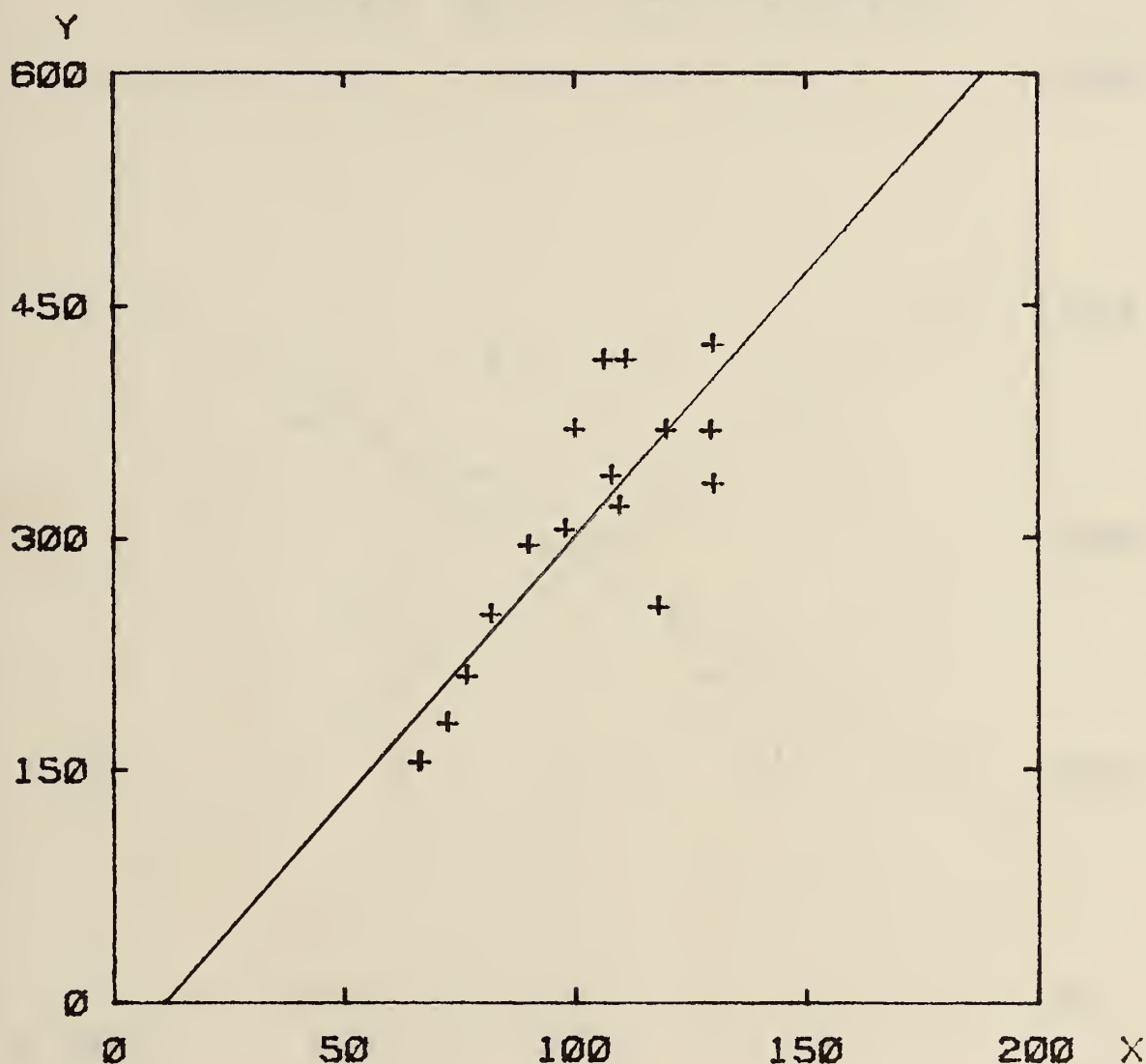
$$Y = -927.4367 + 6.1579 X$$

$$R\text{-SQUARED} = 0.5443$$

PROJECTIONS	Y	X
1985	514.1349	234.1000
2000	689.0202	262.5000

FIGURE F-38. - Scatter diagram of aluminum demand for dissipative use and U.S. population.

ALUMINUM DEMAND--DISSIPATIVE
THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION
1967 = 100

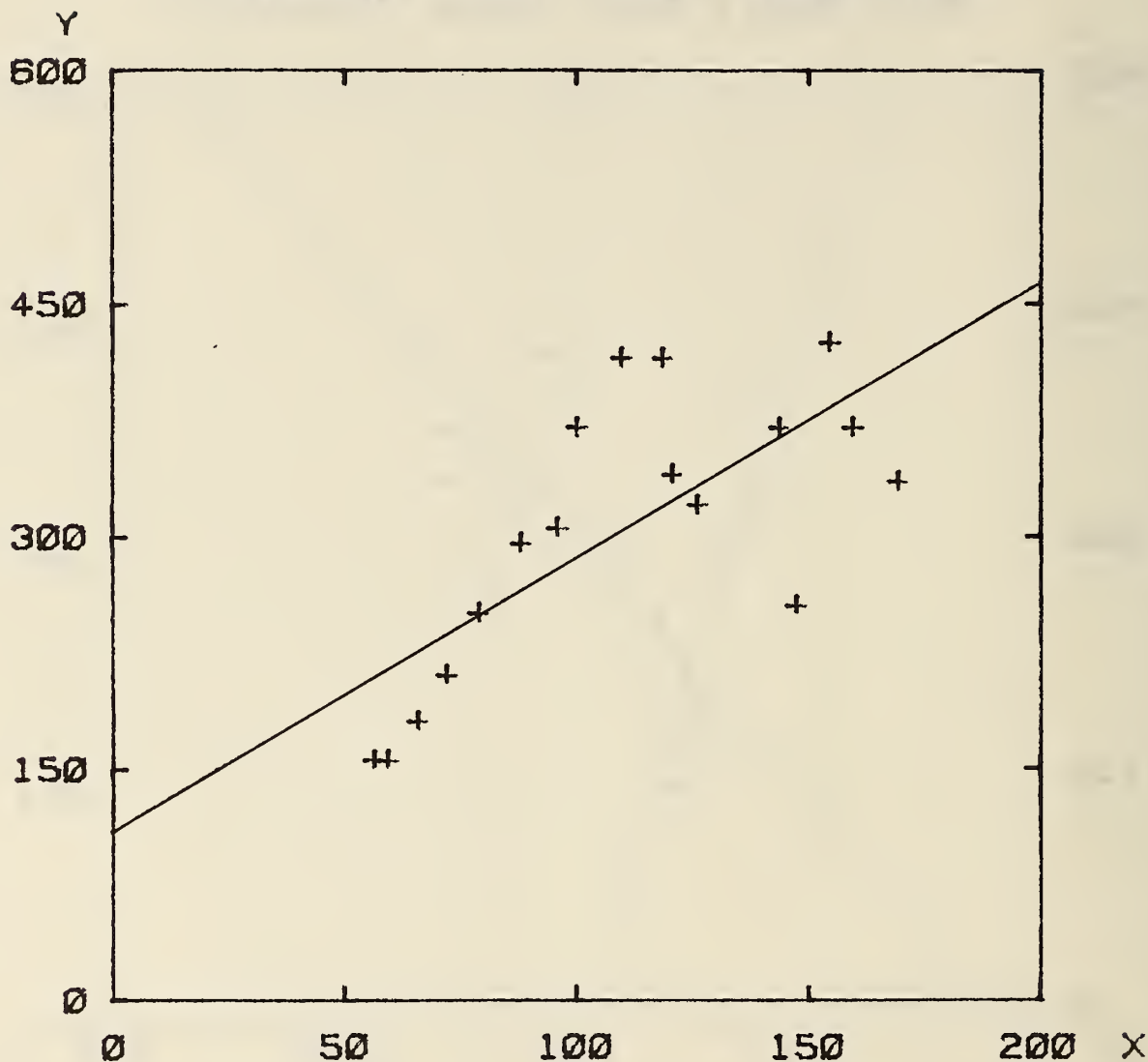
$$Y = -37.5917 + 3.3900 X$$

R-SQUARED = 0.6836

PROJECTIONS	Y	X
1985	593.9696	186.3000
2000	1013.3155	310.0000

FIGURE F-39. - Scatter diagram of aluminum demand for dissipative use and FRB index for total production.

ALUMINUM DEMAND--DISSIPATIVE
THOUSAND SHORT TONS (1960-1976)



FRBI-CHEMICALS AND PRODUCTS
1967 = 100

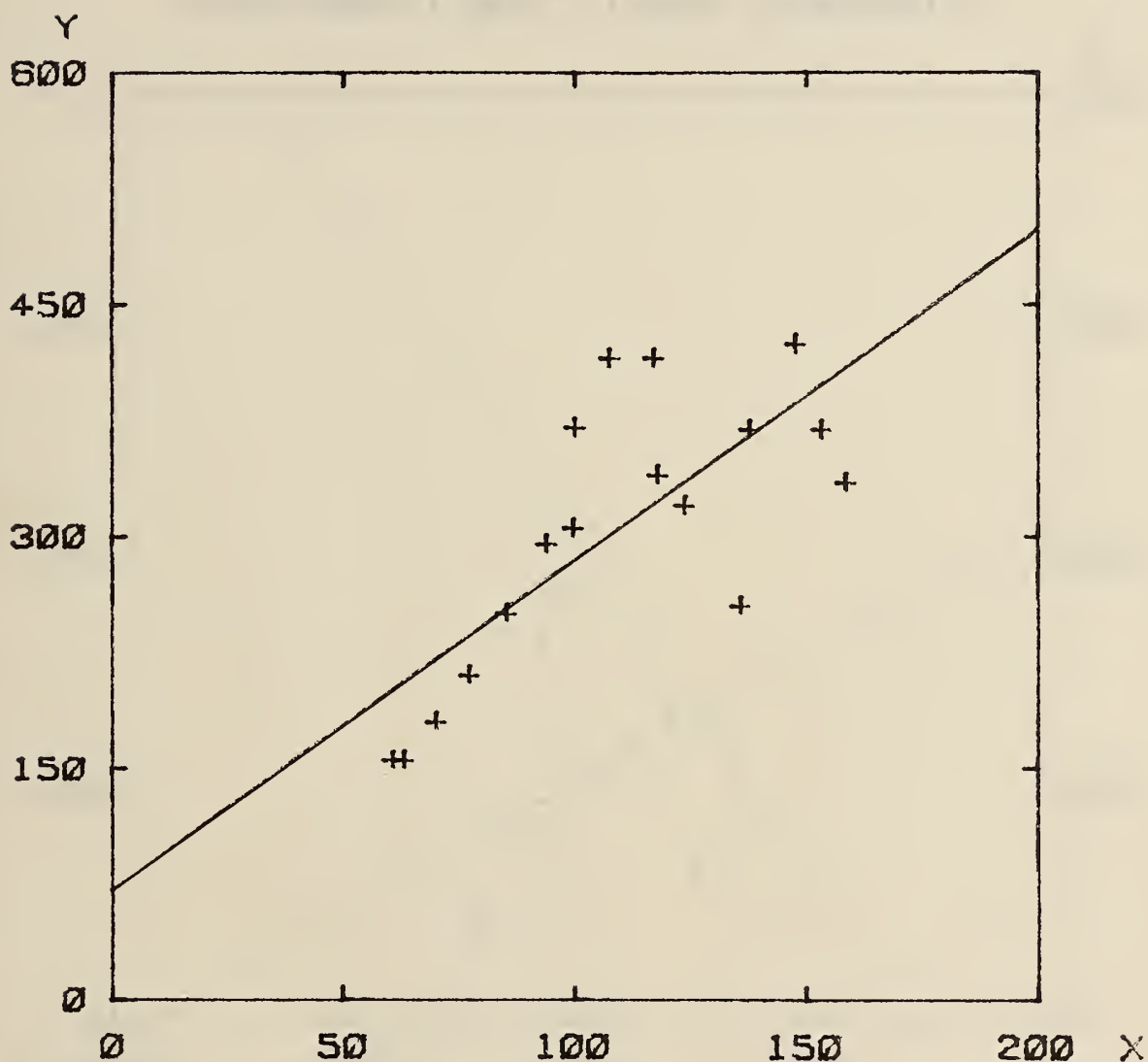
$$Y = 109.1964 + 1.7753 X$$

$$R\text{-SQUARED} = 0.5267$$

PROJECTIONS	Y	X
1985	628.4770	292.5000
2000	1248.9507	642.0000

FIGURE F-40. - Scatter diagram of aluminum demand for dissipative use and FRB index for chemicals and products.

ALUMINUM DEMAND--DISSIPATIVE
THOUSAND SHORT TONS (1960-1976)



FRBI-BASIC CHEMICALS
1967 = 100

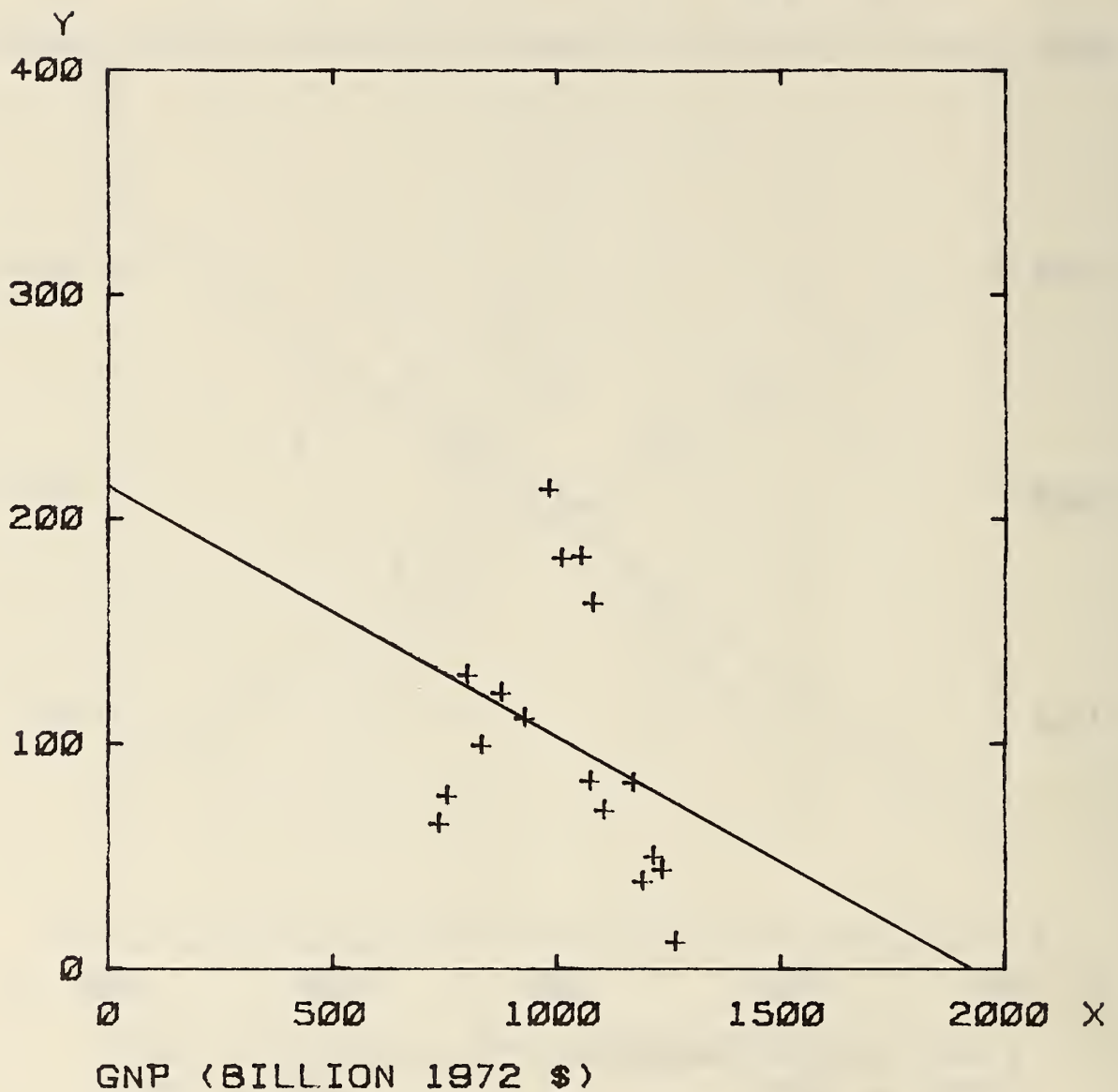
$$Y = 71.2801 + 2.1378 X$$

$$R\text{-SQUARED} = 0.5572$$

PROJECTIONS	Y	X
1985	537.9639	218.3000
2000	787.4463	335.0000

FIGURE F-41. - Scatter diagram of aluminum demand for dissipative use and FRB index for basic chemicals.

ALUMINUM DEMAND--OTHER METAL
THOUSAND SHORT TONS (1960-1976)



$$Y = 214.6663 - 0.1114 X$$

$$R-SQUARED = 0.1135$$

PROJECTIONS

1985

17.4239

1770.5000

2000

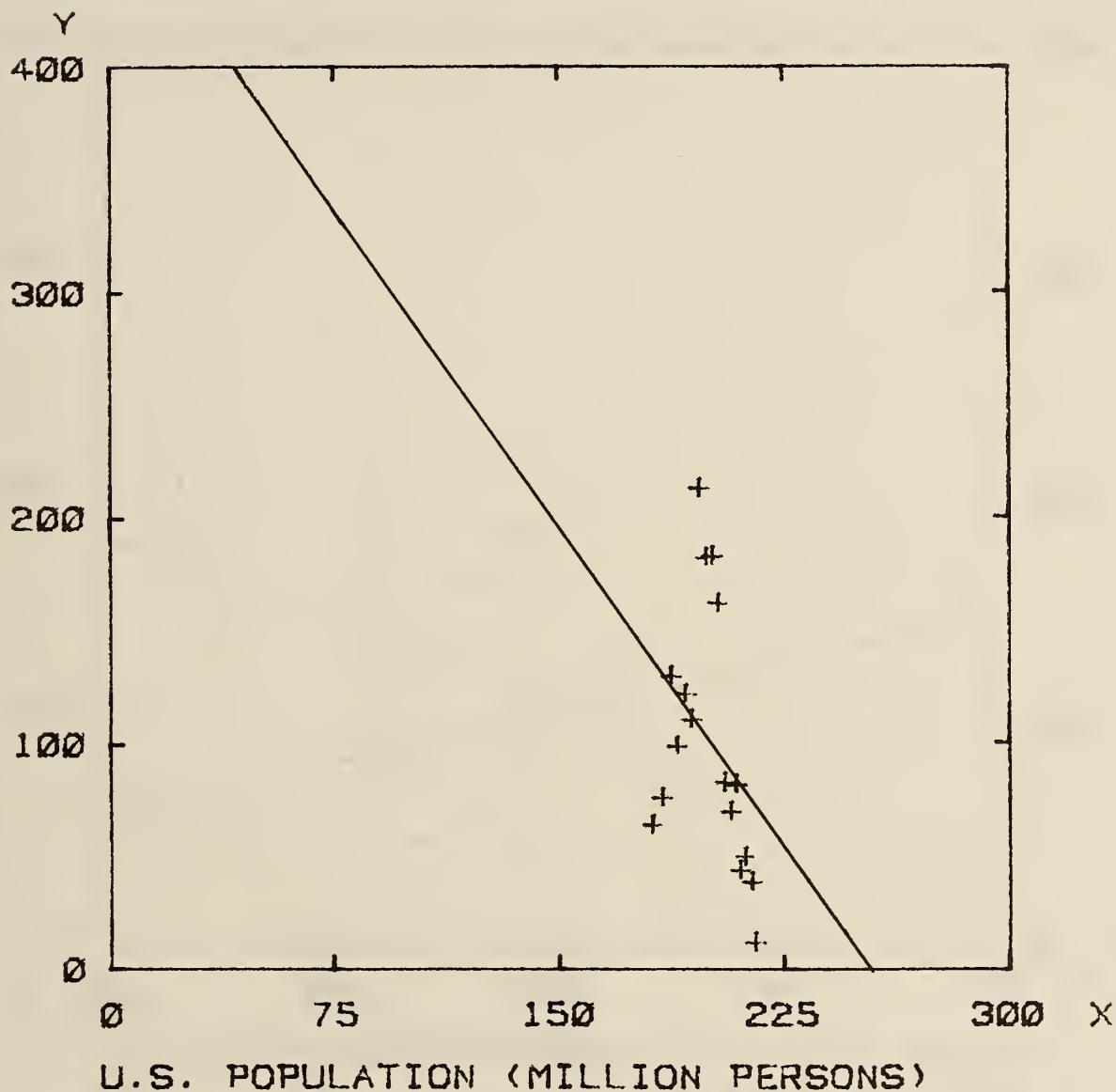
-

96.3762

2792.0000

FIGURE F-42. - Scatter diagram of aluminum demand for other metal use and gross national product (GNP).

ALUMINUM DEMAND--OTHER METAL
THOUSAND SHORT TONS (1960-1976)



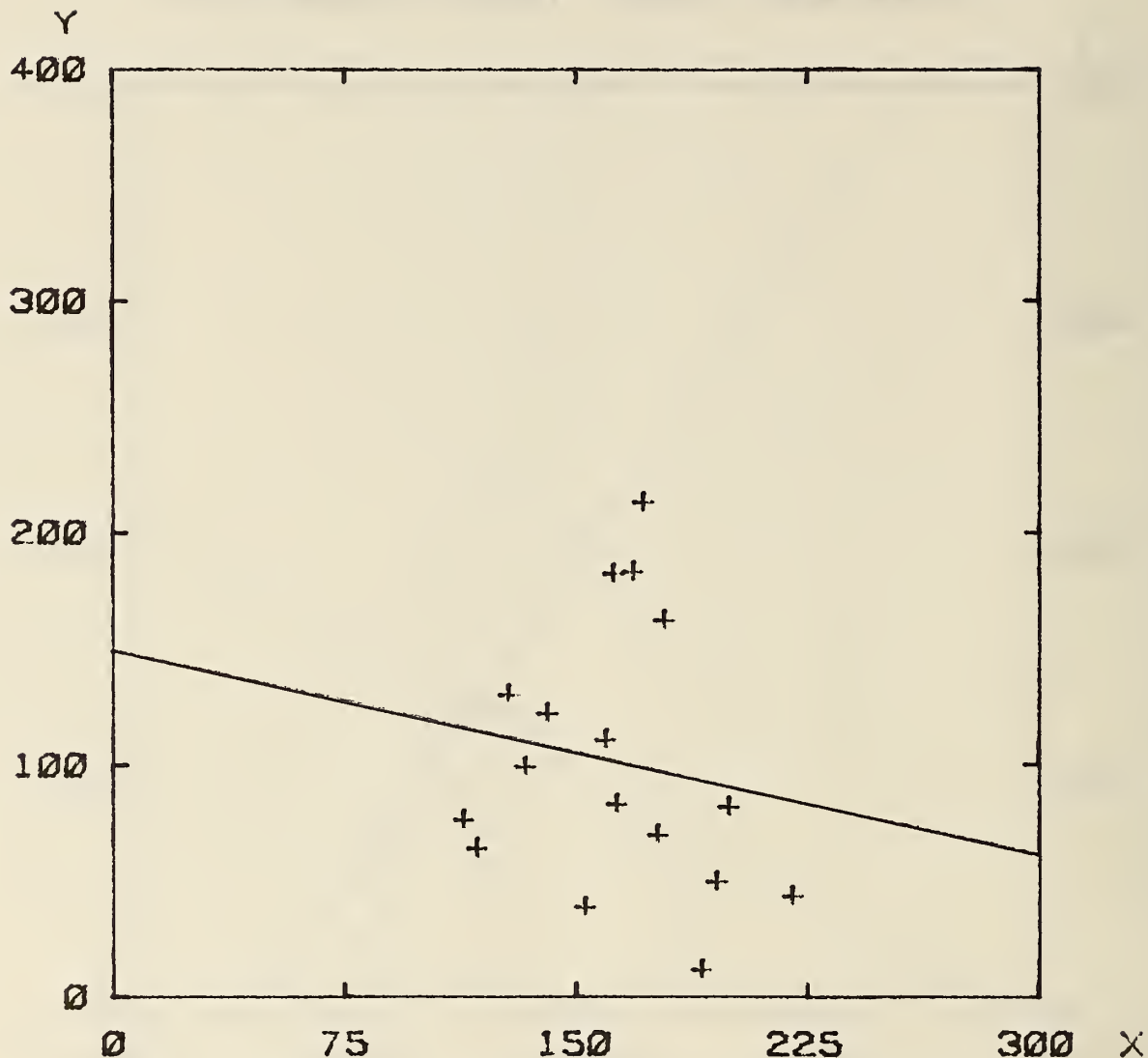
$Y = 479.9729 - 1.8938 X$

$R\text{-SQUARED} = 0.1271$

PROJECTIONS	Y	X
1985	36.6120	234.1000
2000	- 17.1746	262.5000

FIGURE F-43. - Scatter diagram of aluminum demand for other metal use and U.S. population.

ALUMINUM DEMAND--OTHER METAL
THOUSAND SHORT TONS (1960-1976)



GROSS PRIVATE DOMESTIC INVESTMENT
BILLION 1973 \$

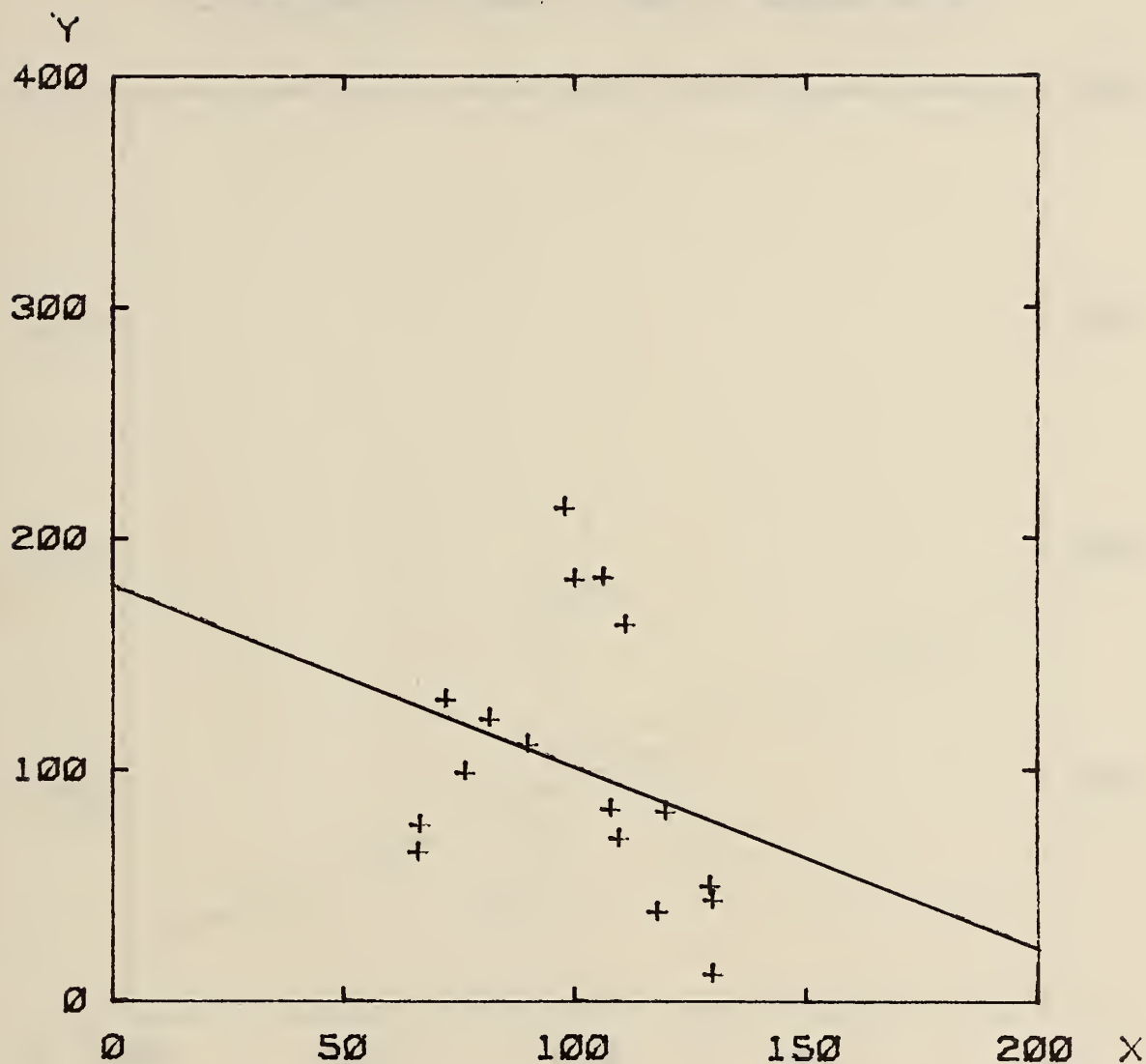
$$Y = 148.9546 + - 0.2925 X$$

$$R\text{-SQUARED} = 0.0231$$

PROJECTIONS	Y	X
1985	57.1052	314.0000
2000	3.1526	520.0000

FIGURE F-44. - Scatter diagram of aluminum demand for other metal use and gross private domestic investment.

ALUMINUM DEMAND--OTHER METAL
THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION
1967 = 100

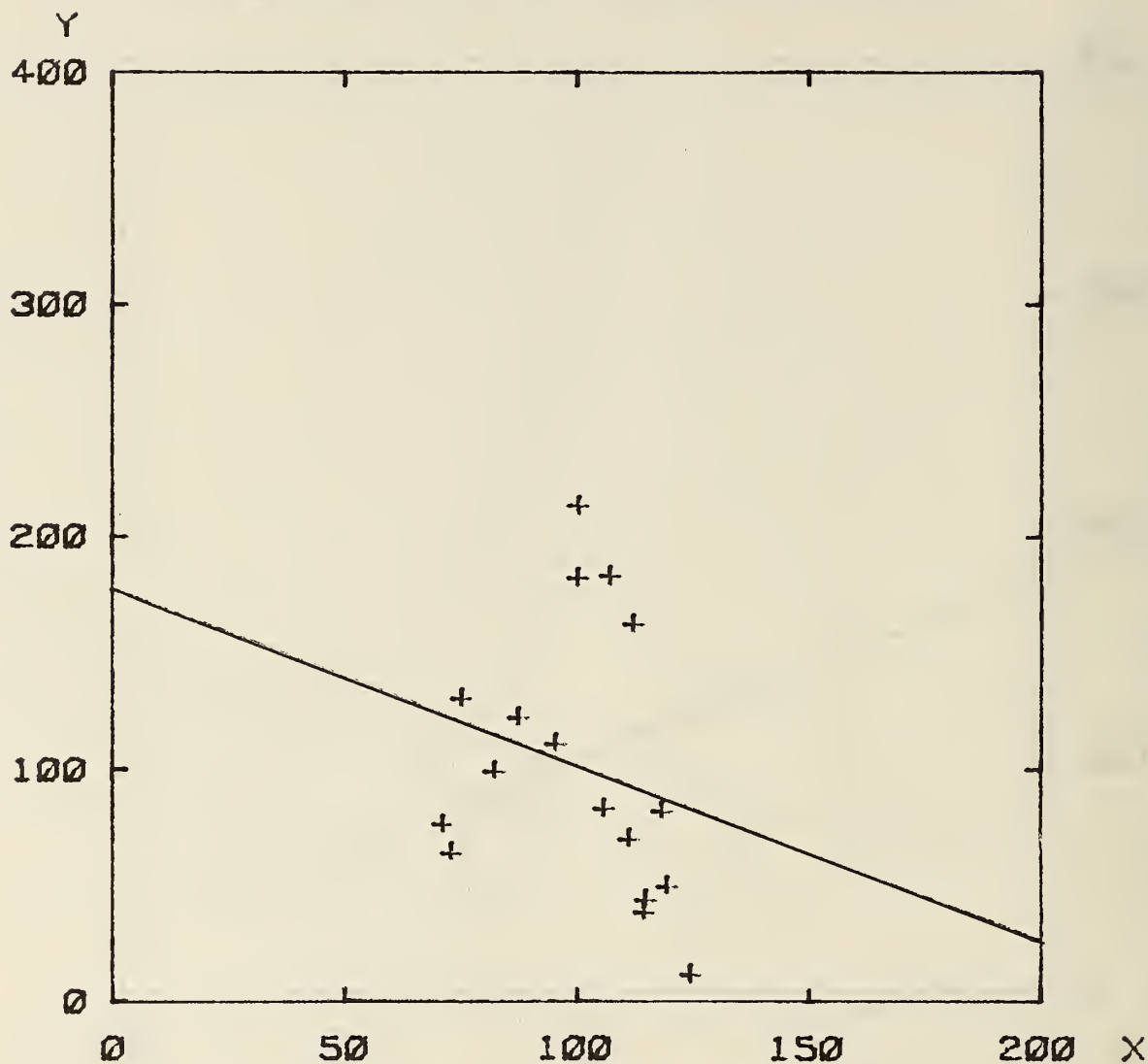
$$Y = 179.8416 + - 0.7799 X$$

$$R\text{-SQUARED} = 0.0893$$

PROJECTIONS	Y	X
1985	34.5418	186.3000
2000	- 61.9347	310.0000

FIGURE F-45. - Scatter diagram of aluminum demand for other metal use and FRB index for total production.

ALUMINUM DEMAND--OTHER METAL
THOUSAND SHORT TONS (1960-1976)



FRBI-PAINTS
1967 = 100

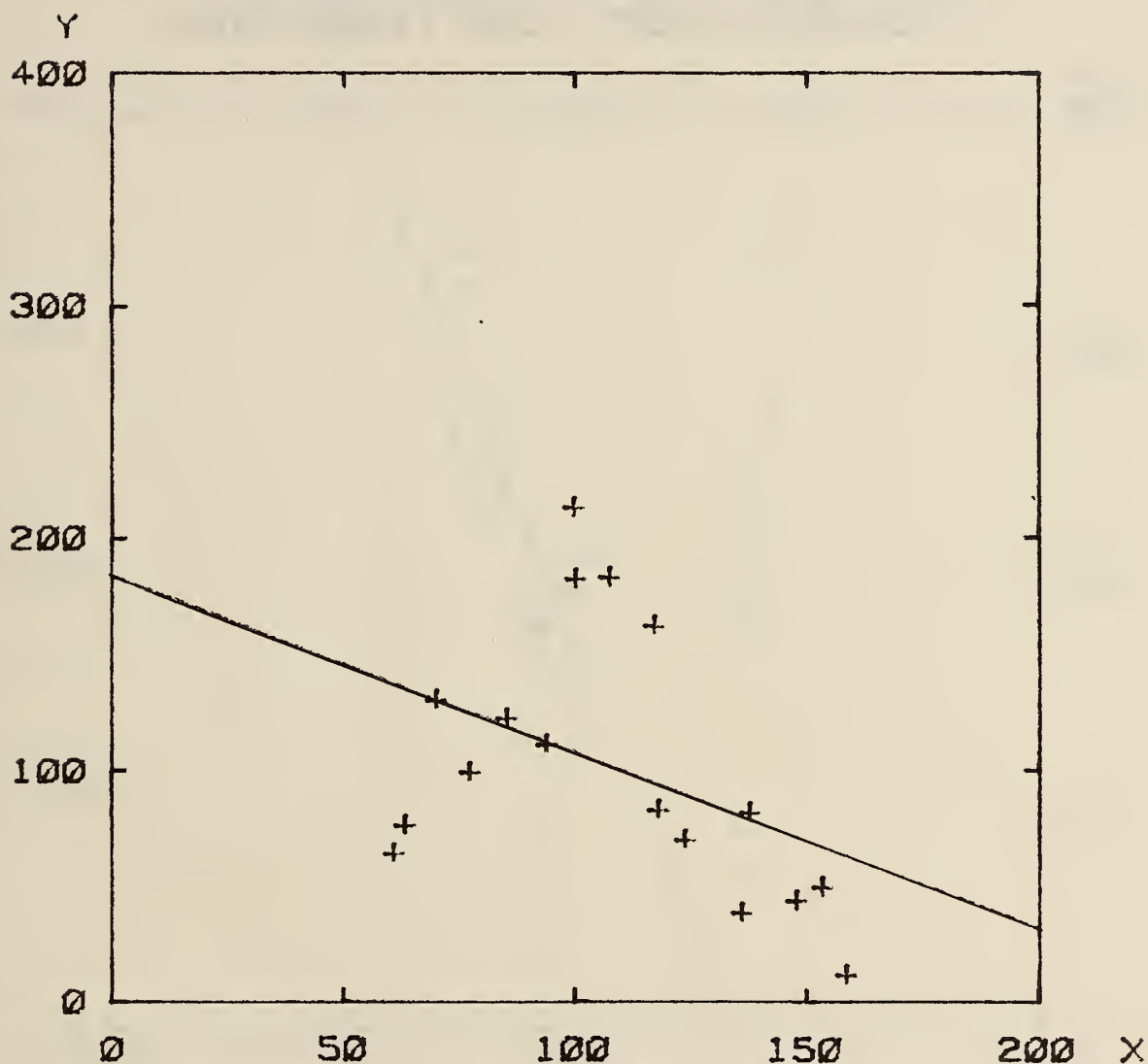
$$Y = 177.1799 - 0.7544 X$$

$$R\text{-SQUARED} = 0.0516$$

PROJECTIONS	Y	X
1985	25.0798	201.6000
2000	58.5903	312.5000

FIGURE F-46. - Scatter diagram of aluminum demand for other metal use and FRB index for paints.

ALUMINUM DEMAND--OTHER METAL
THOUSAND SHORT TONS (1960-1976)



FRBI-BASIC CHEMICALS
1967 = 100

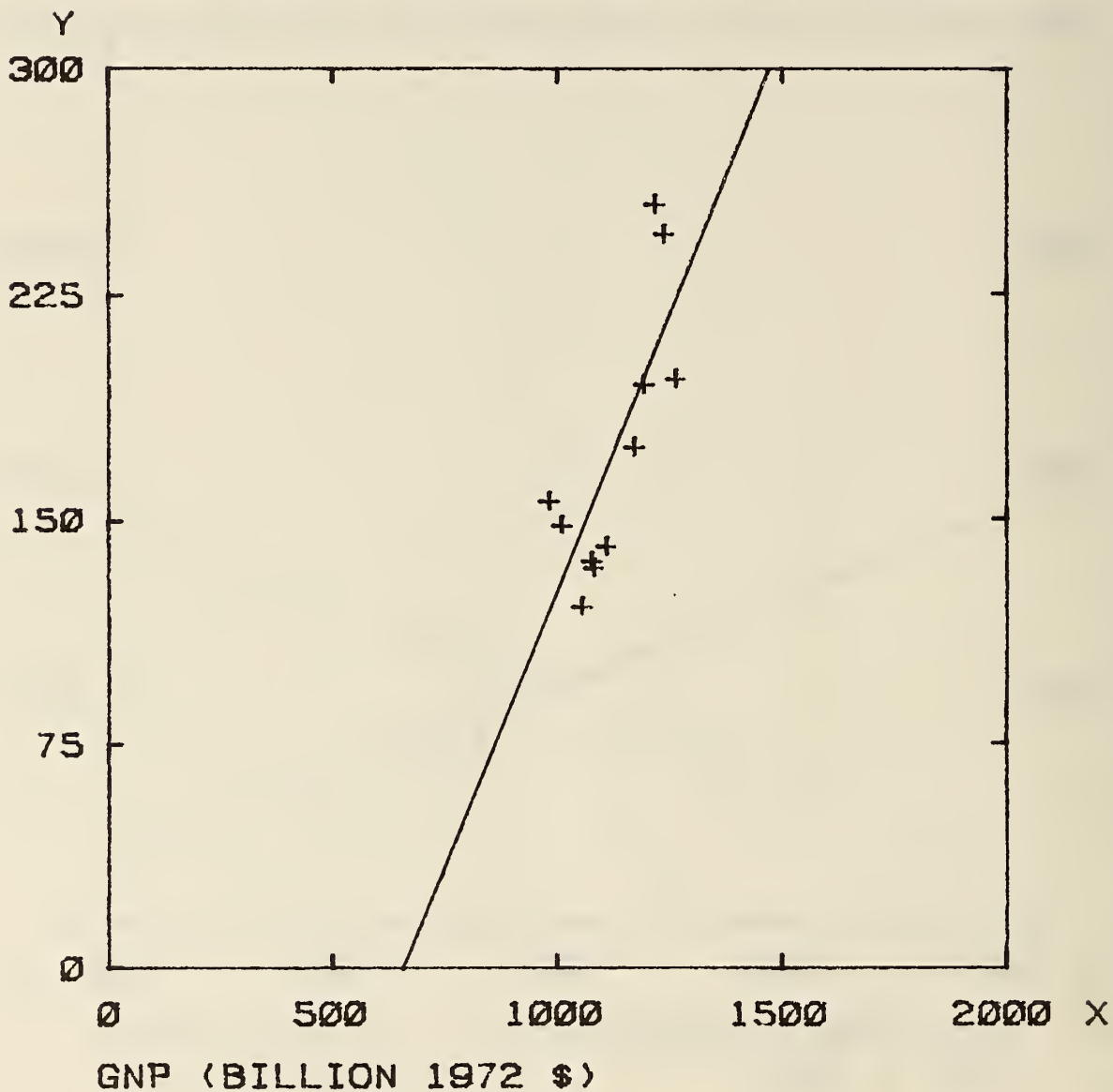
$$Y = 183.7706 - 0.7582 X$$

$$R\text{-SQUARED} = 0.1731$$

PROJECTIONS	Y	X
1985	18.2511	218.3000
2000	- 70.2331	335.0000

FIGURE F-47. - Scatter diagram of aluminum demand for other metal use and FRB index for basic chemicals.

ALUMINUM DEMAND--REFRACTORIES
THOUSAND SHORT TONS (1960-1976)



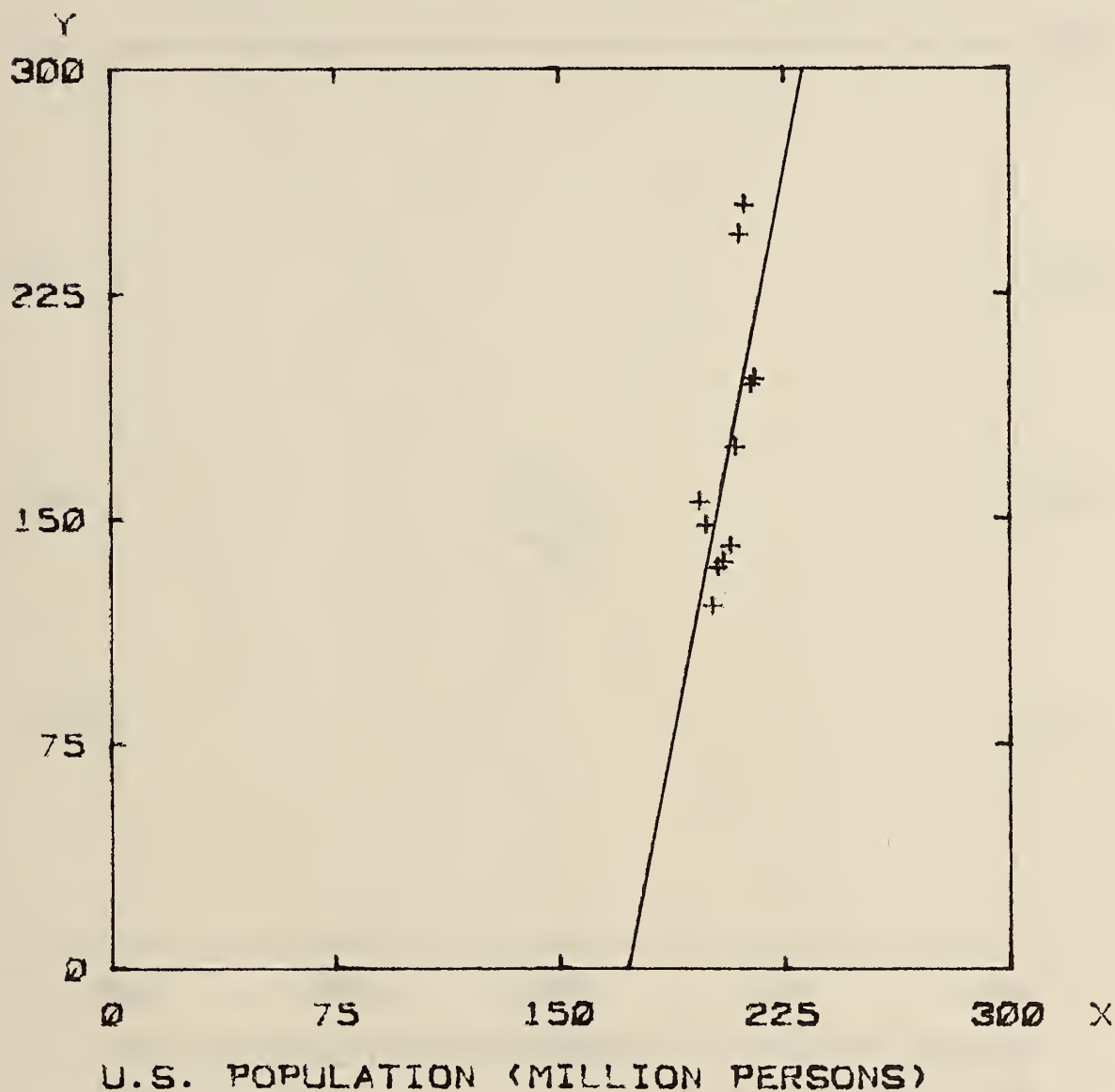
$$Y = -241.7664 + 0.3685 X$$

$$R\text{-SQUARED} = 0.6014$$

PROJECTIONS	Y	X
1985	410.6825	1770.5000
2000	787.1167	2792.0000

FIGURE F-48. - Scatter diagram of aluminum demand for refractories use and gross national product (GNP).

ALUMINUM DEMAND--REFRACTORIES
THOUSAND SHORT TONS (1960-1976)



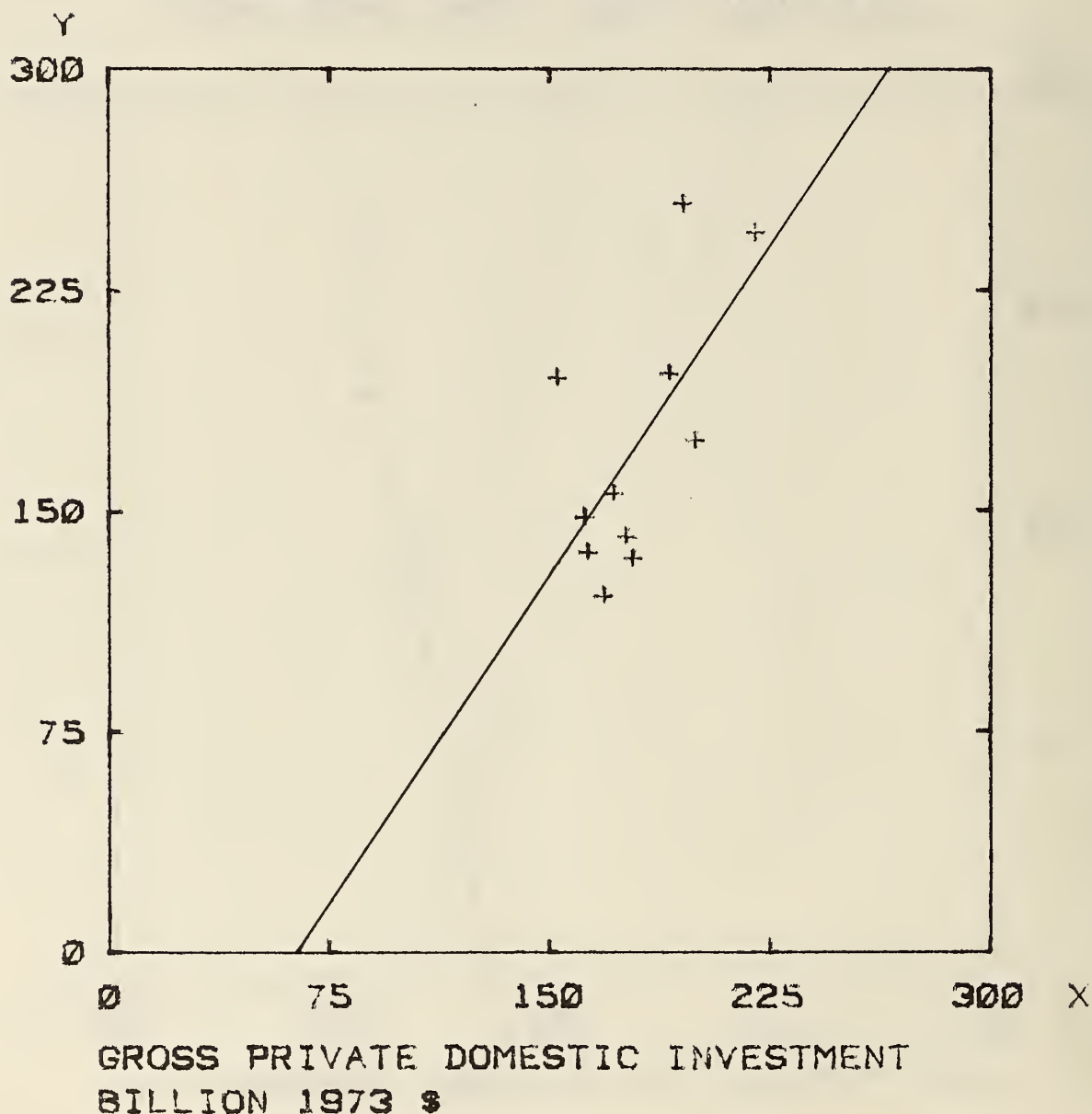
$$Y = - 879.8362 + 5.0966 X$$

$$R\text{-SQUARED} = 0.4693$$

PROJECTIONS	Y	X
1985	313.2998	234.1000
2000	458.0459	262.5000

FIGURE F-49. - Scatter diagram of aluminum demand for refractories use and U.S. population.

ALUMINUM DEMAND--REFRACTORIES
THOUSAND SHORT TONS (1960-1976)

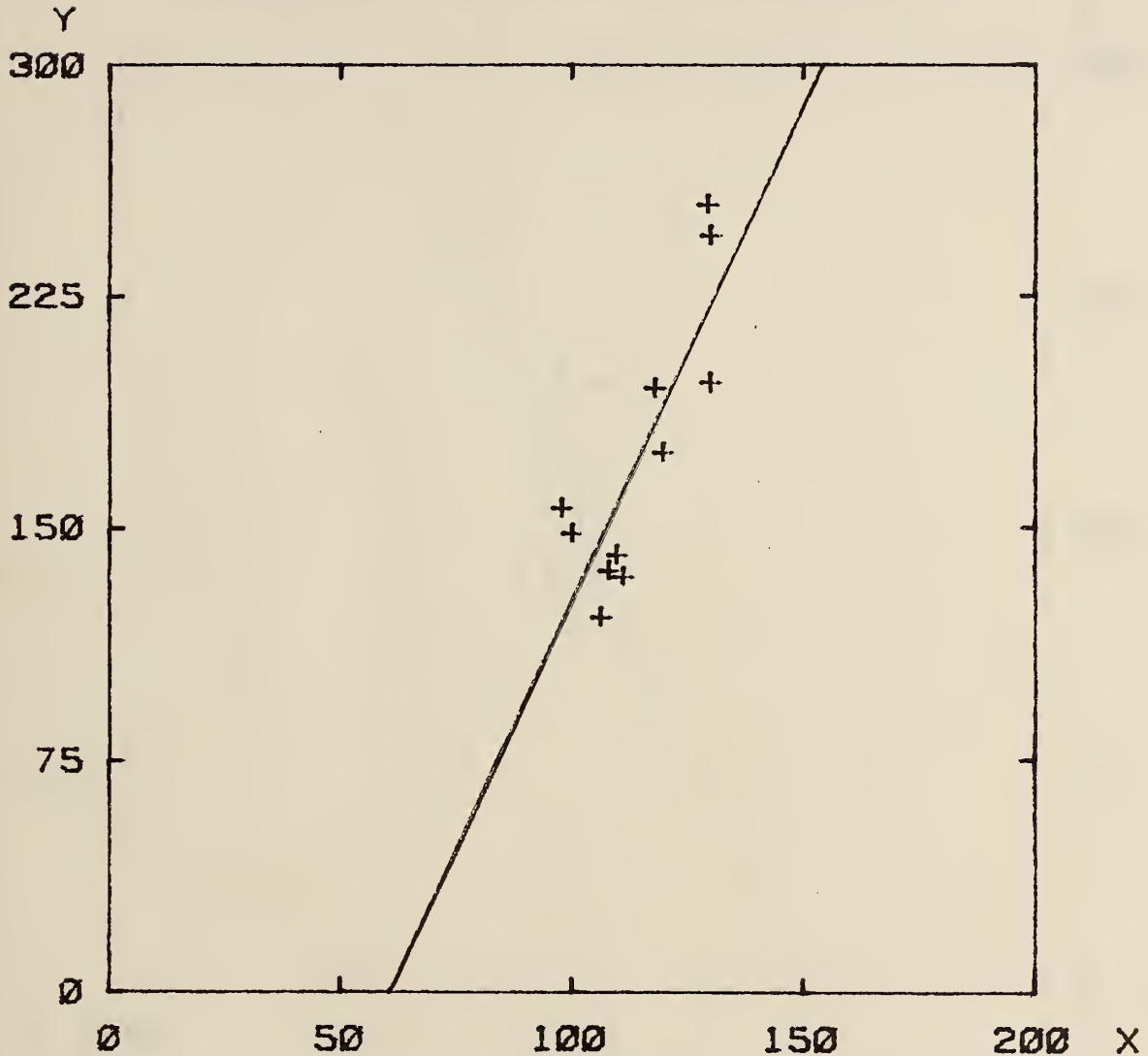


PROJECTIONS

	Y	X
1985	373.2155	314.0000
2000	680.6591	520.0000

FIGURE F-50. - Scatter diagram of aluminum demand for refractories use and gross private domestic investment.

ALUMINUM DEMAND--REFRACTORIES
THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION
1967 = 100

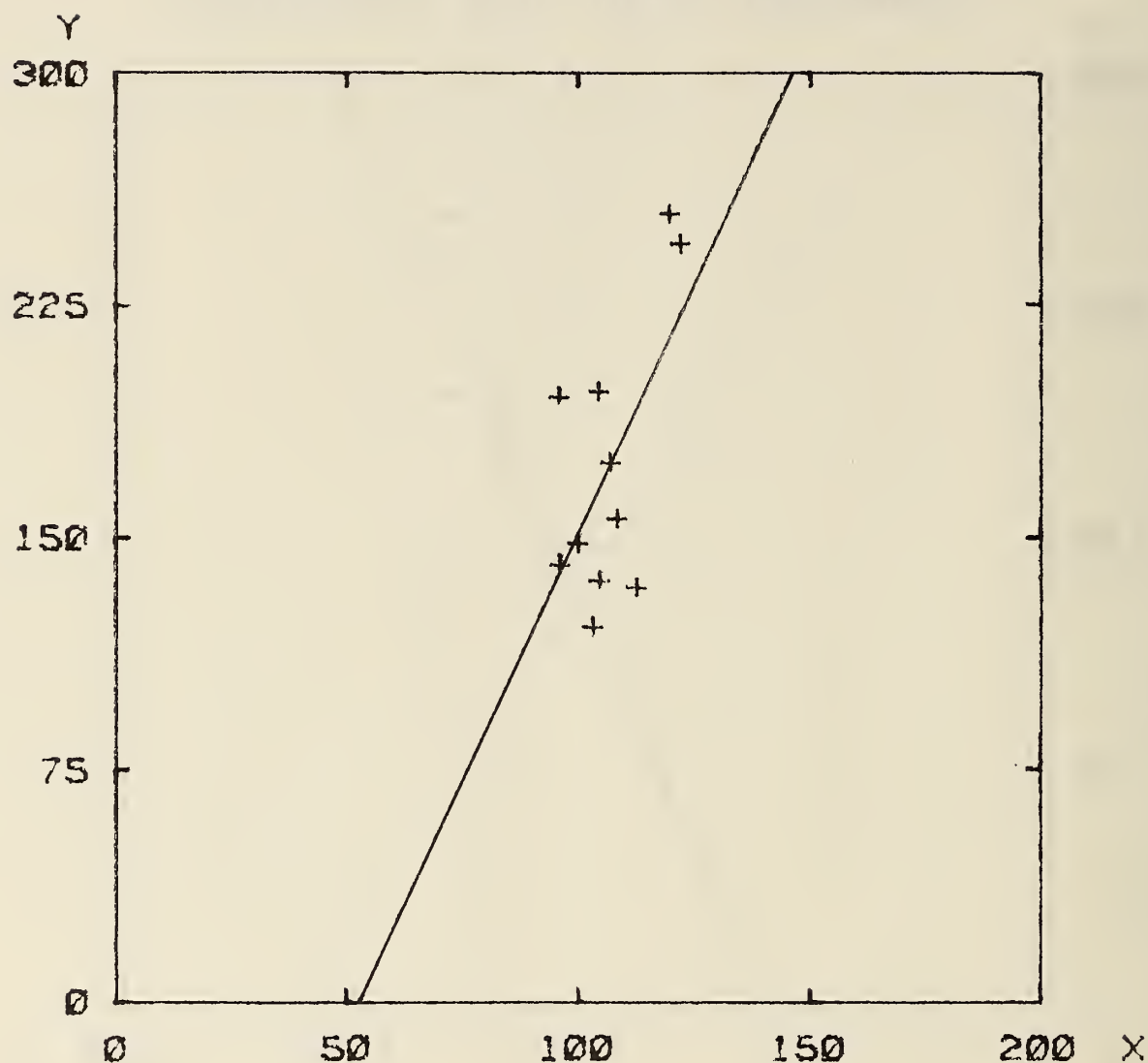
$$Y = -194.3504 + 3.2087 X$$

R-SQUARED = 0.6840

PROJECTIONS	Y	X
1985	403.4454	186.3000
2000	800.3716	310.0000

FIGURE F-51. - Scatter diagram of aluminum demand for refractories use and FRB index for total production.

ALUMINUM DEMAND--REFRACTORIES
THOUSAND SHORT TONS (1966-1976)



FRBI-IRON AND STEEL
1967 = 100

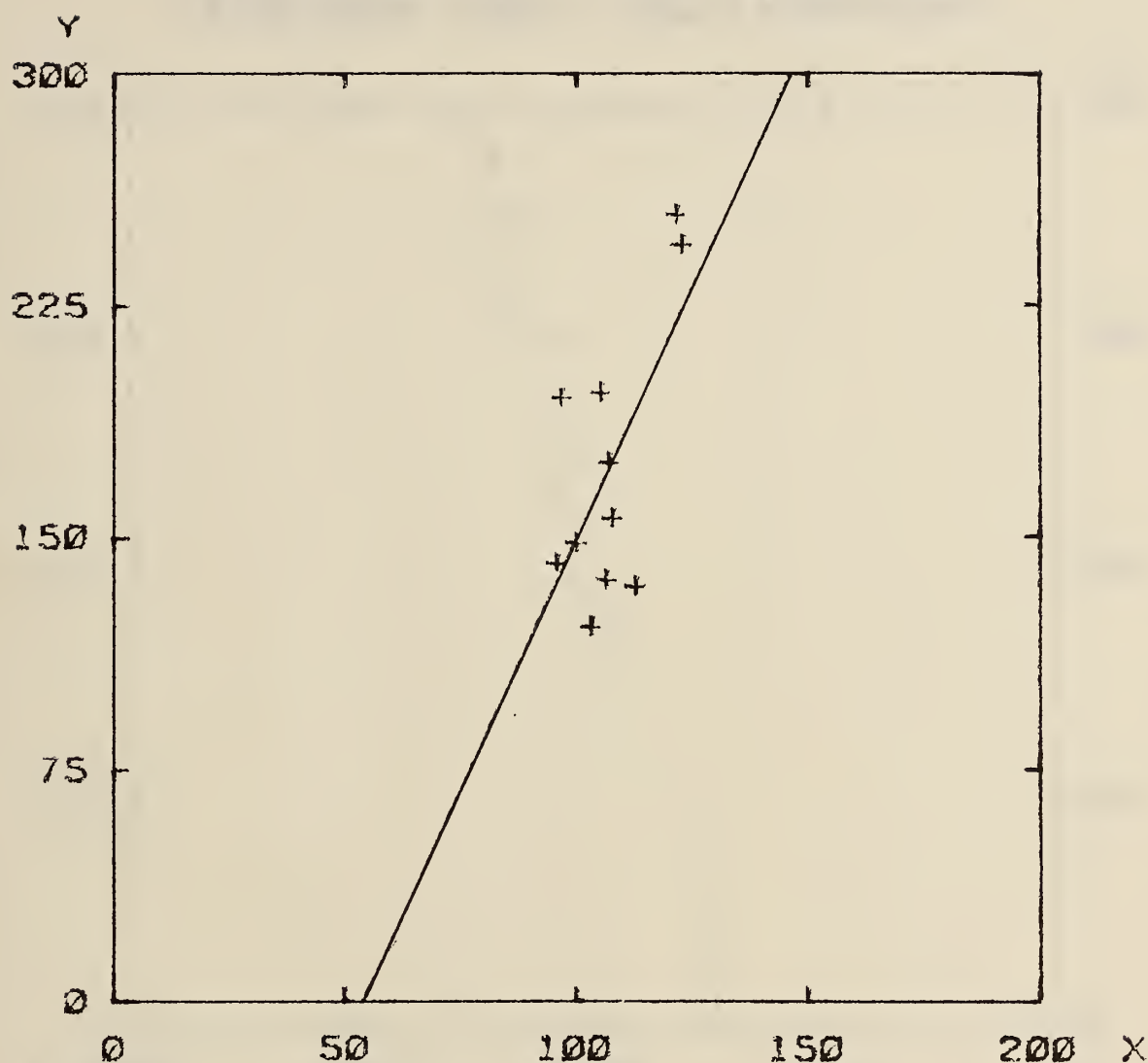
$$Y = -169.4929 + 3.2071 X$$

$$R\text{-SQUARED} = 0.3764$$

PROJECTIONS	Y	X
1985	332.4187	156.5000
2000	484.7561	204.0000

FIGURE F-52. - Scatter diagram of aluminum demand for refractories use and FRB index for iron and steel.

ALUMINUM DEMAND--REFRACTORIES THOUSAND SHORT TONS (1866-1976)



FRBI-BASIC STEEL AND MILL PRODUCTS
1967 = 100

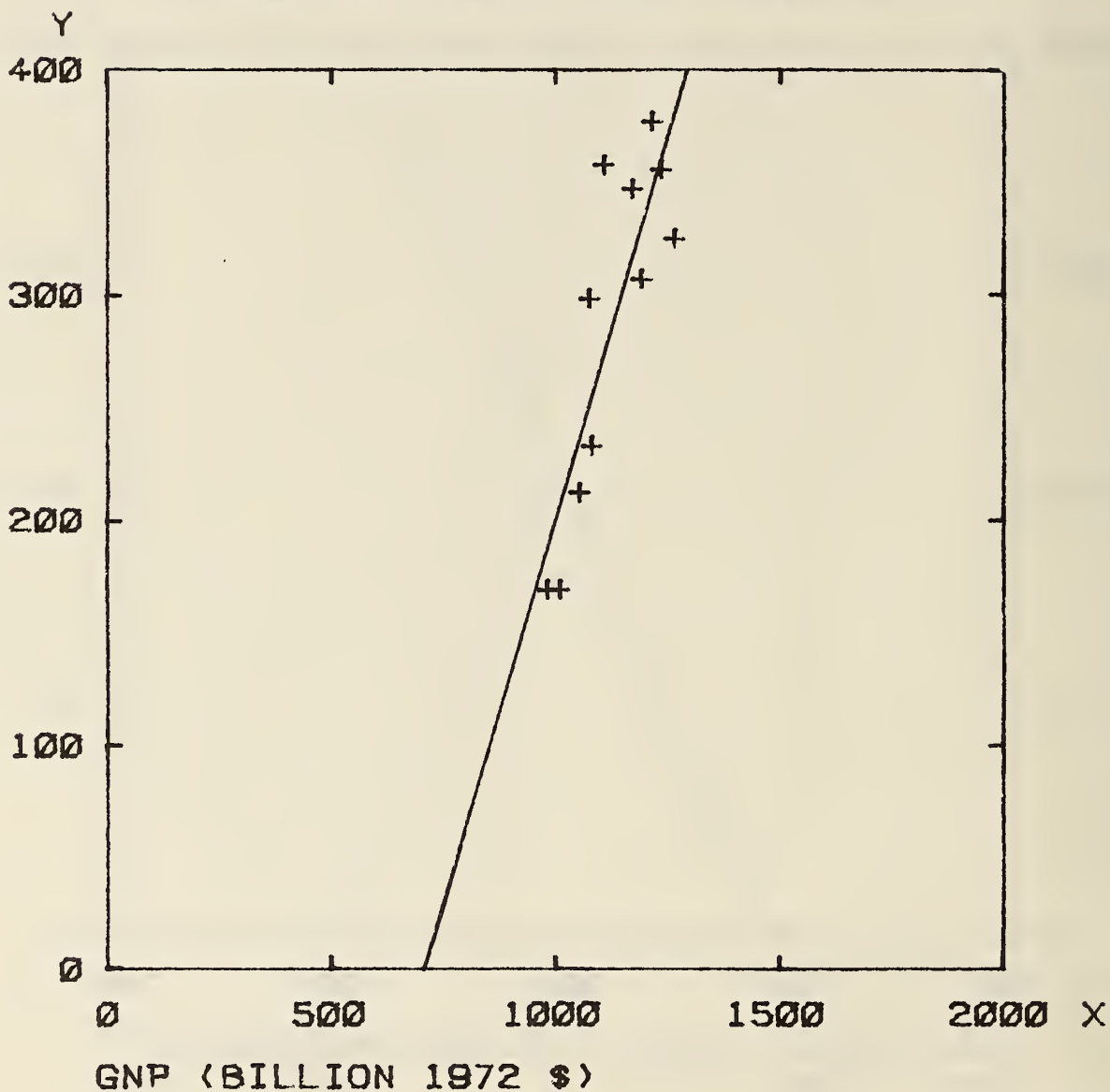
$$Y = -175.5767 + 3.2483 X$$

$$R\text{-SQUARED} = 0.4083$$

PROJECTIONS	Y	X
1985	337.0083	157.8000
2000	509.4942	210.9000

FIGURE F-53. - Scatter diagram of aluminum demand for refractories use and FRB index for basic steel and mill products.

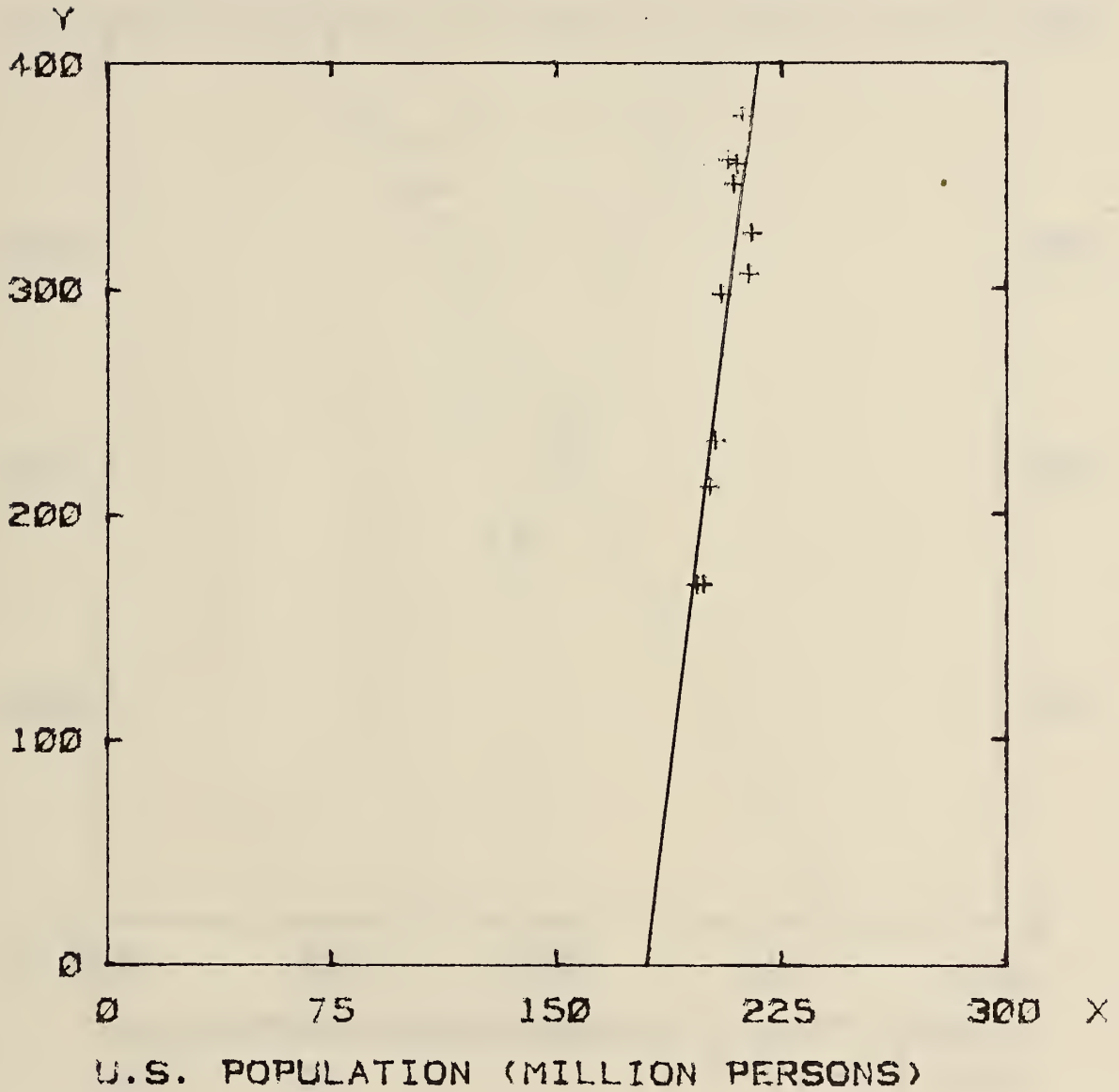
ALUMINUM DEMAND--CHEMICALS
THOUSAND SHORT TONS (1960-1976)



PROJECTIONS	Y	X
1985	727.2460	1770.5000
2000	1425.0908	2792.0000

FIGURE F-54. - Scatter diagram of aluminum demand for chemicals use and gross national product (GNP).

ALUMINUM DEMAND--CHEMICALS THOUSAND SHORT TONS (1960-1976)

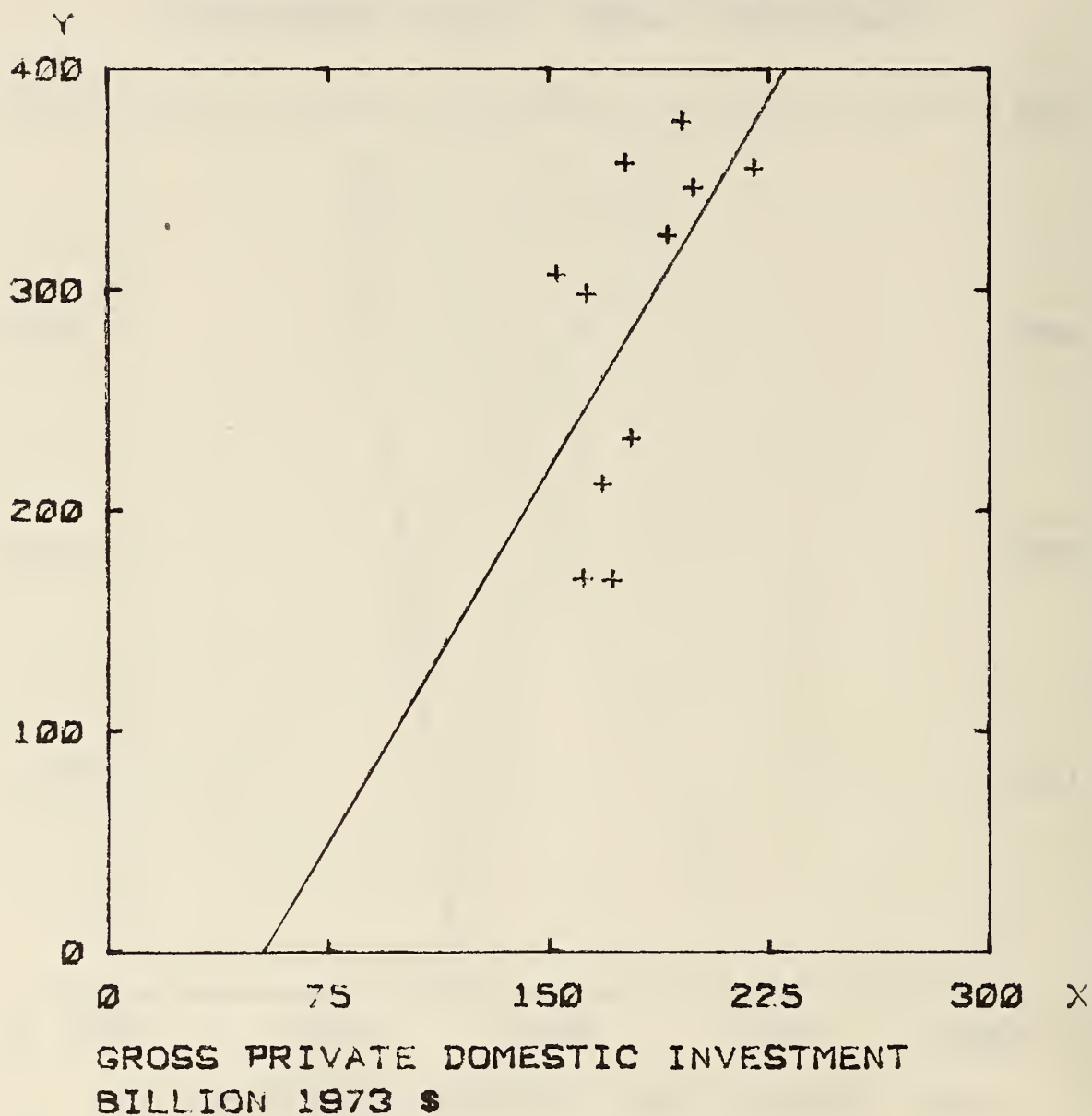


PROJECTIONS

	Y	X
1985	583.1656	234.1000
2000	889.0815	262.5000

FIGURE F-55. - Scatter diagram of aluminum demand for chemicals use and U.S. population.

ALUMINUM DEMAND--CHEMICALS
THOUSAND SHORT TONS (1960-1976)



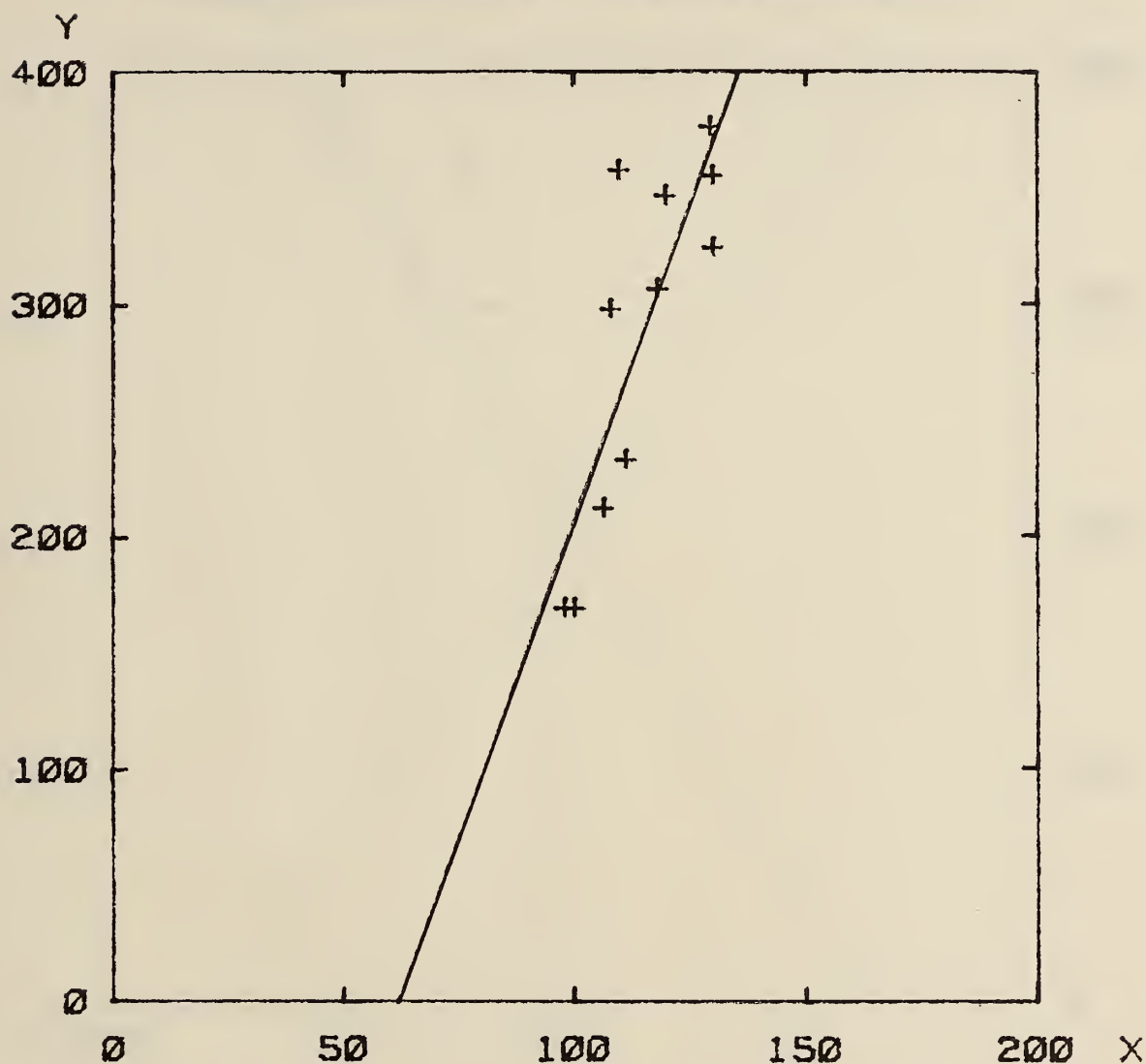
$$Y = -119.1208 + 2.2558 X$$

$$R\text{-SQUARED} = 0.3319$$

PROJECTIONS	Y	X
1985	589.2236	314.0000
2000	1053.9337	520.0000

FIGURE F-56. - Scatter diagram of aluminum demand for chemicals use and gross private domestic investment.

ALUMINUM DEMAND--CHEMICALS
THOUSAND SHORT TONS (1960-1976)



FRBI-TOTAL PRODUCTION
1967 = 100

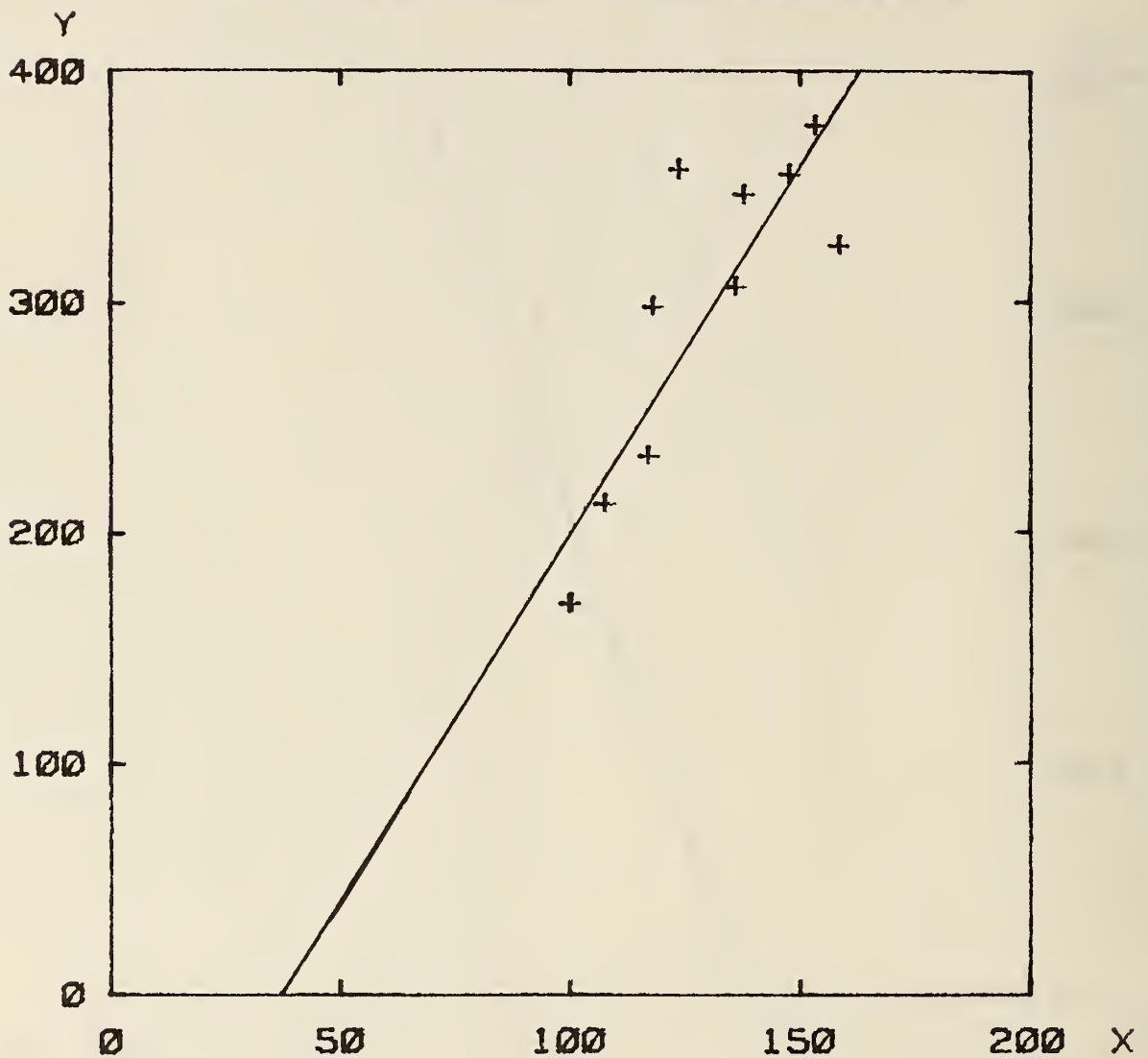
$$Y = -335.8693 + 5.4373 X$$

$$R\text{-SQUARED} = 0.6727$$

PROJECTIONS	Y	X
1985	677.0999	186.3000
2000	1349.6942	310.0000

FIGURE F-57. - Scatter diagram of aluminum demand for chemicals use and FRB index for total production.

ALUMINUM DEMAND--CHEMICALS
THOUSAND SHORT TONS (1960-1976)



FRBI-BASIC CHEMICALS
1967 = 100

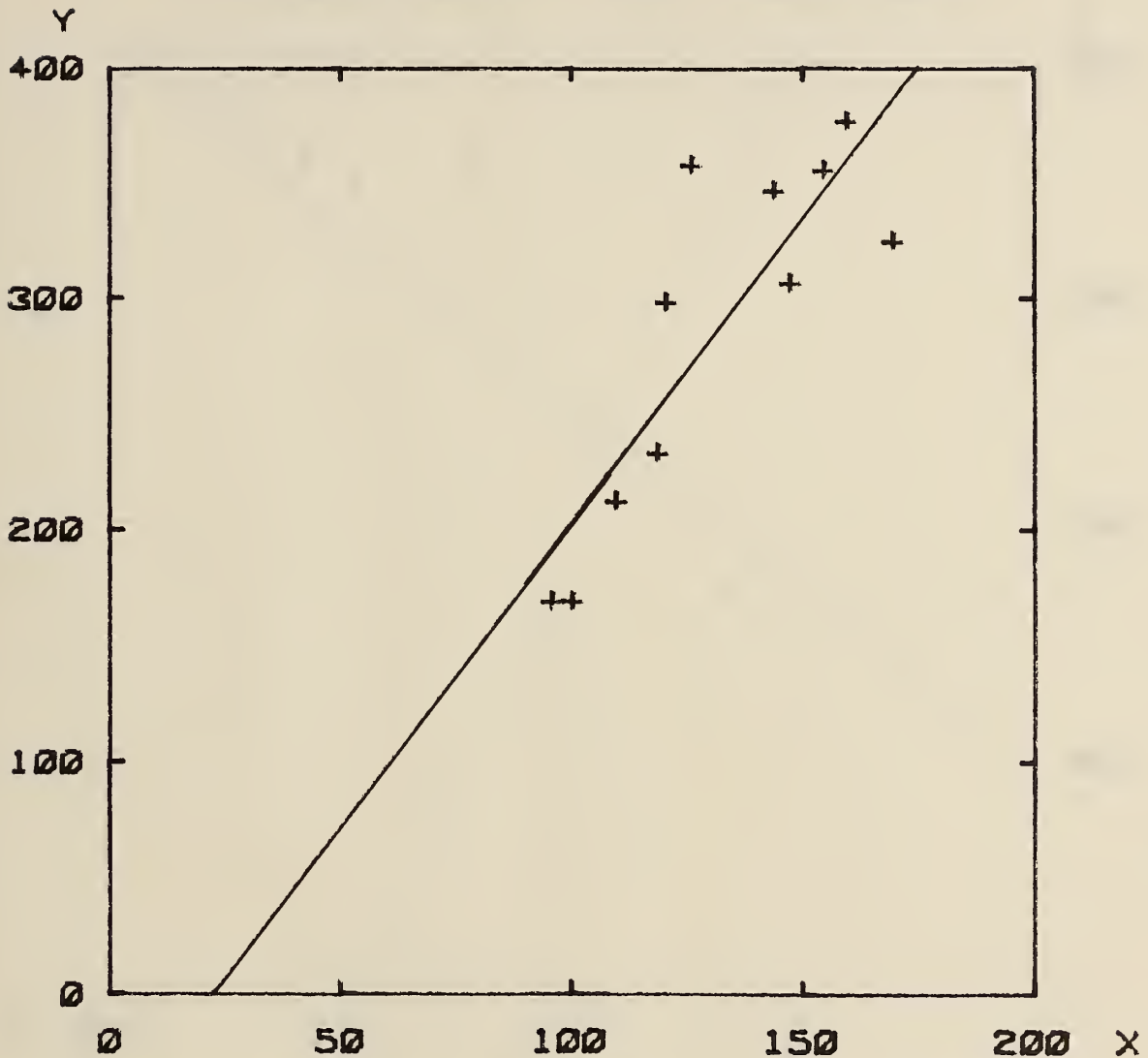
$$Y = -118.8334 + 3.1873 X$$

$$R-SQUARED = 0.7406$$

PROJECTIONS	Y	X
1985	576.9683	218.3000
2000	948.9339	335.0000

FIGURE F-58. - Scatter diagram of aluminum demand for chemicals use and FRB index for basic chemicals.

ALUMINUM DEMAND--CHEMICALS
THOUSAND SHORT TONS (1960-1976)



FRBI-CHEMICALS AND PRODUCTS
1967 = 100

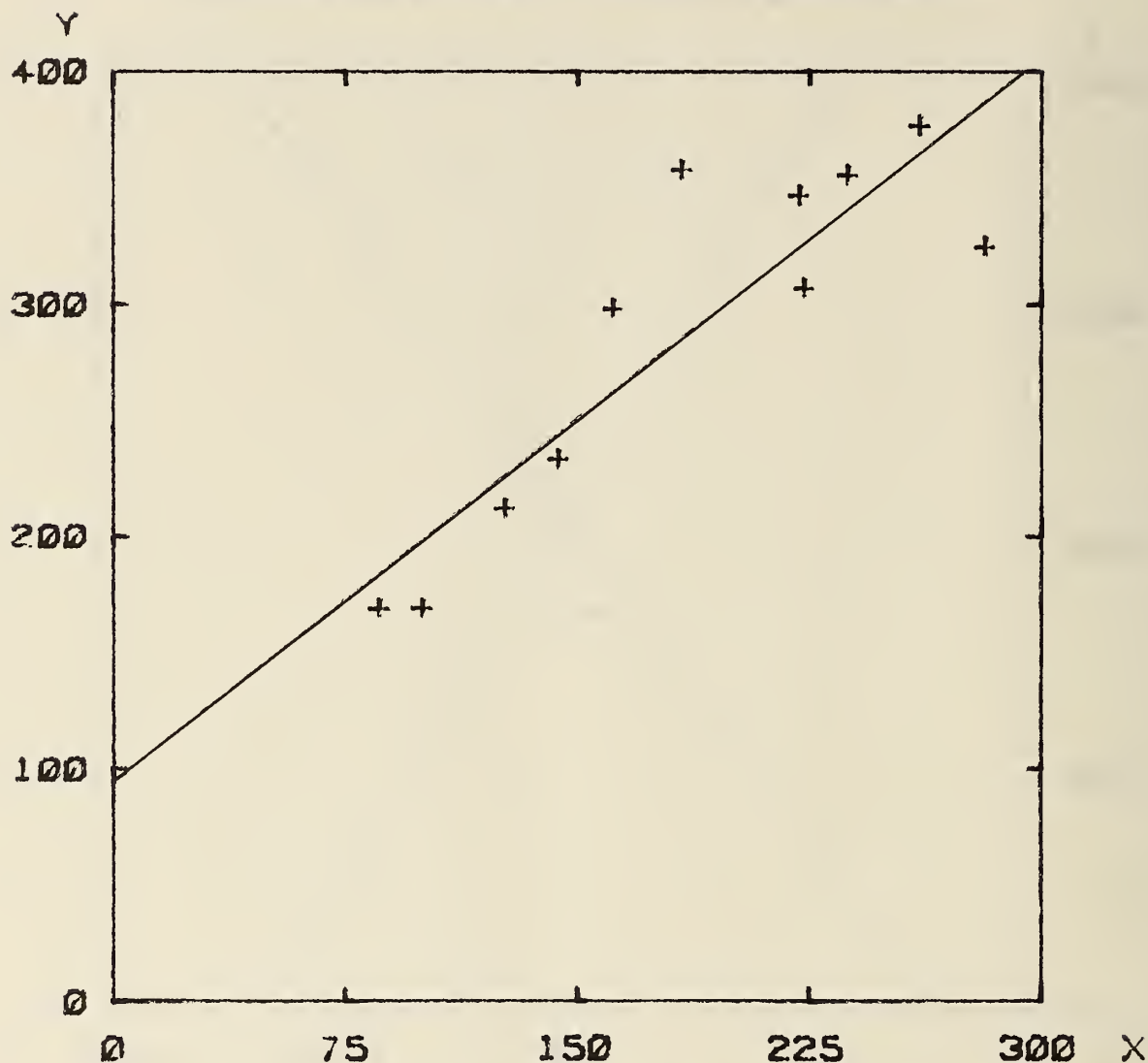
$$Y = - 58.7242 + 2.6294 X$$

$$R-SQUARED = 0.7185$$

PROJECTIONS	Y	X
1985	710.3981	292.5000
2000	1629.4006	642.0000

FIGURE F-59. - Scatter diagram of aluminum demand for chemicals use and FRB index for chemicals and products.

ALUMINUM DEMAND--CHEMICALS
THOUSAND SHORT TONS (1960-1976)



FRBI-PLASTICS PRODUCTS, NEC.
1967 = 100

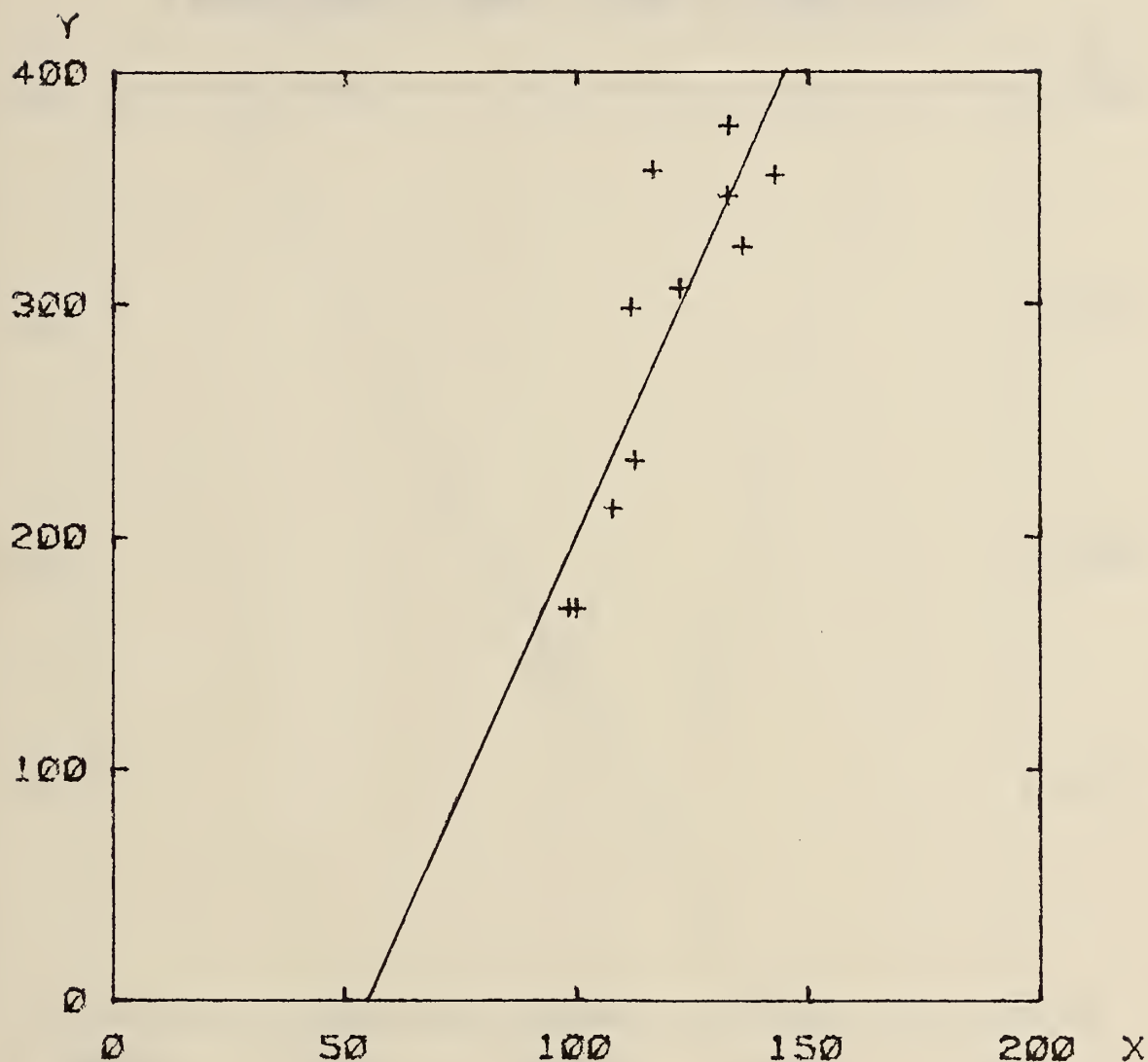
$$Y = 95.0251 + 1.0389 X$$

$$R\text{-SQUARED} = 0.7816$$

PROJECTIONS	Y	X
1985	577.3248	464.2000
2000	1024.2989	894.4000

FIGURE F-60. - Scatter diagram of aluminum demand for chemicals use and FRB index for plastics products, n.e.c.

ALUMINUM DEMAND--CHEMICALS
THOUSAND SHORT TONS (1966-1976)



FRBI-TEXTILE MILL PRODUCTS
1967 = 100

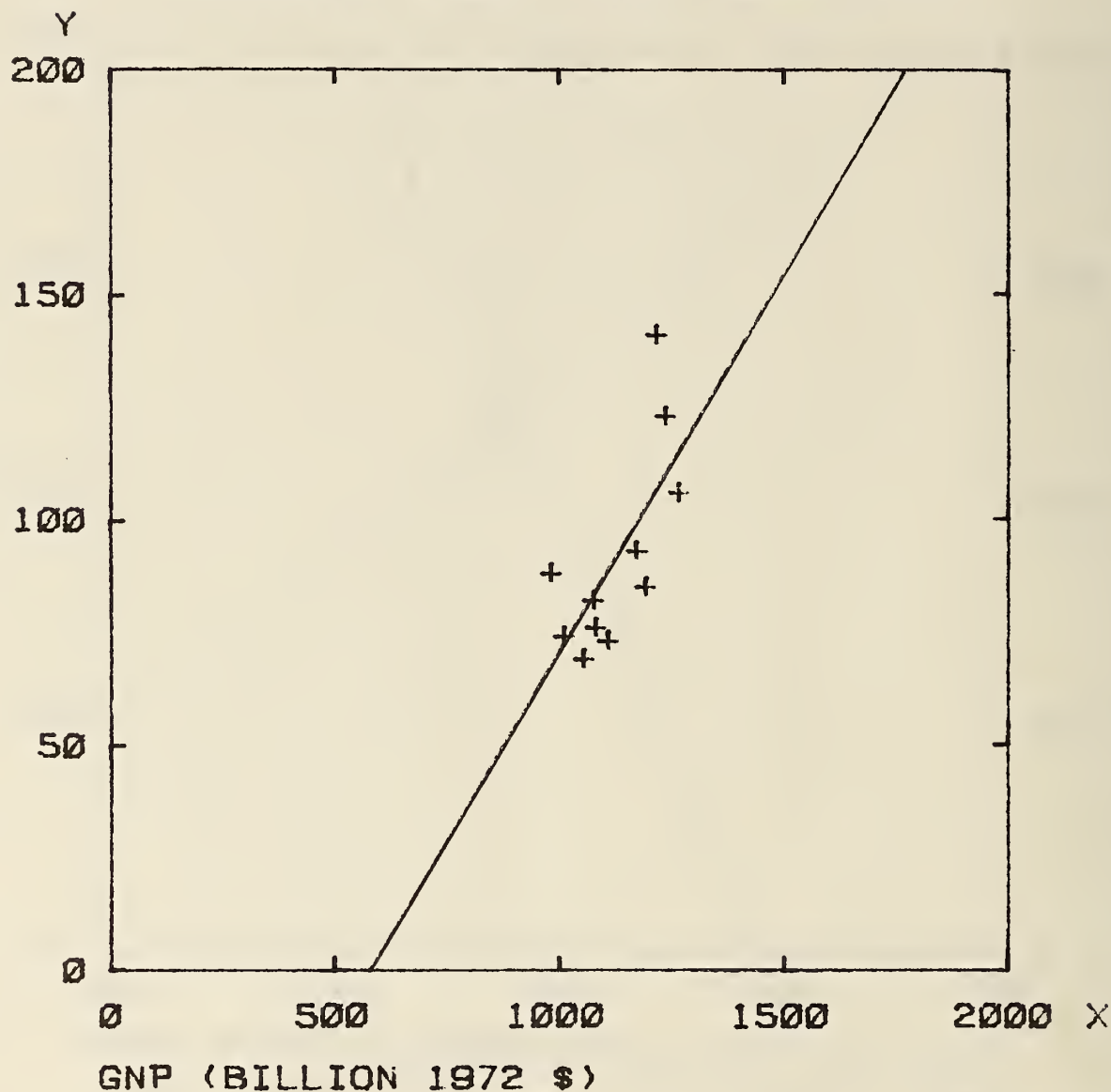
$$Y = -245.0129 + 4.4497 X$$

R-SQUARED = 0.7448

PROJECTIONS	Y	X
1985	614.2427	193.1000
2000	1103.2753	303.0000

FIGURE F-61. - Scatter diagram of aluminum demand for chemicals use and FRB index for textile mill products.

ALUMINUM DEMAND--ABRASIVES
THOUSAND SHORT TONS (1960-1976)



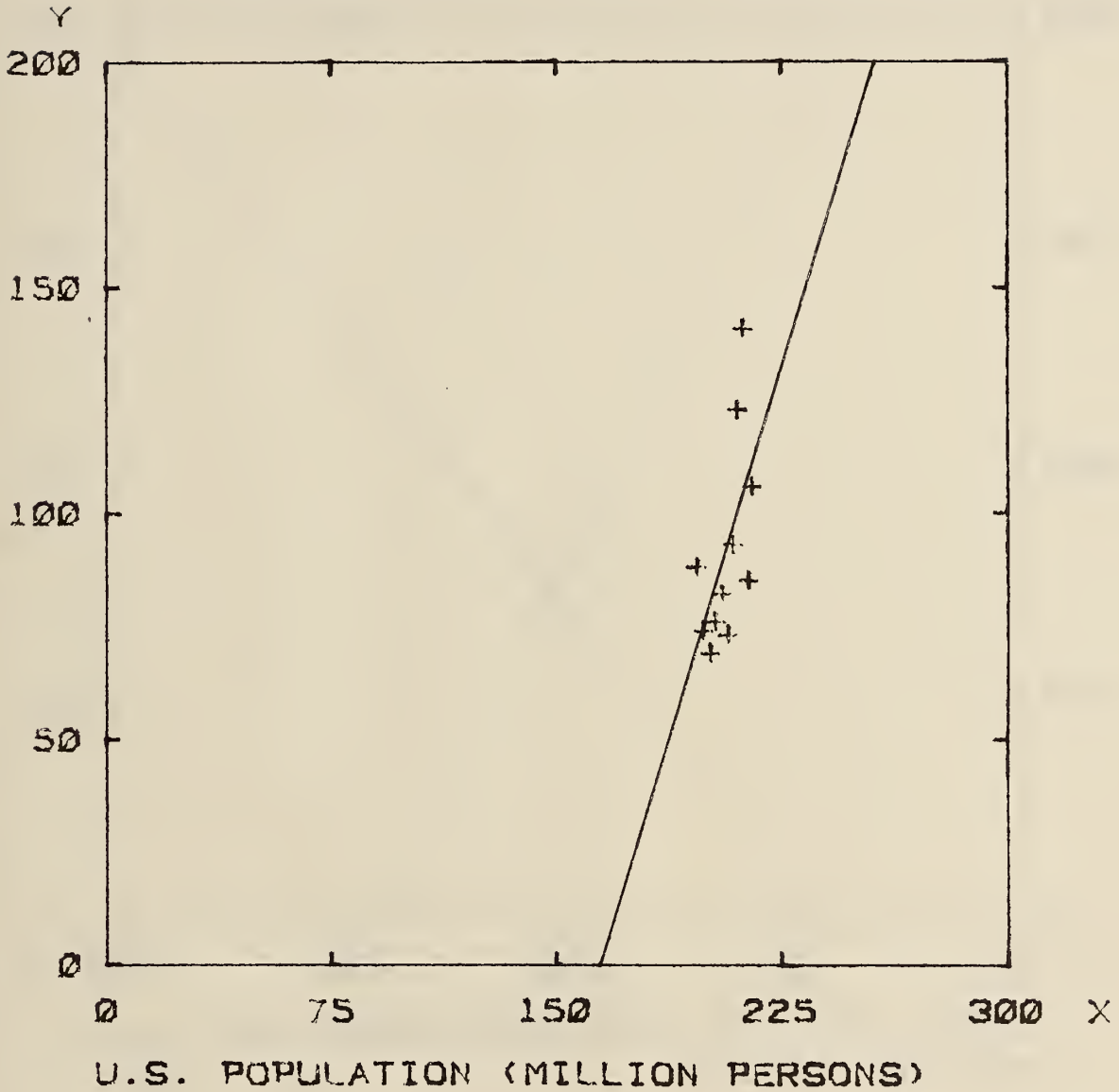
$Y = -97.1890 + 0.1679 X$

$R-SQUARED = 0.4932$

PROJECTIONS	Y	X
1985	200.1942	1770.5000
2000	371.7711	2792.0000

FIGURE F-62. - Scatter diagram of aluminum demand for abrasives use and gross national product (GNP).

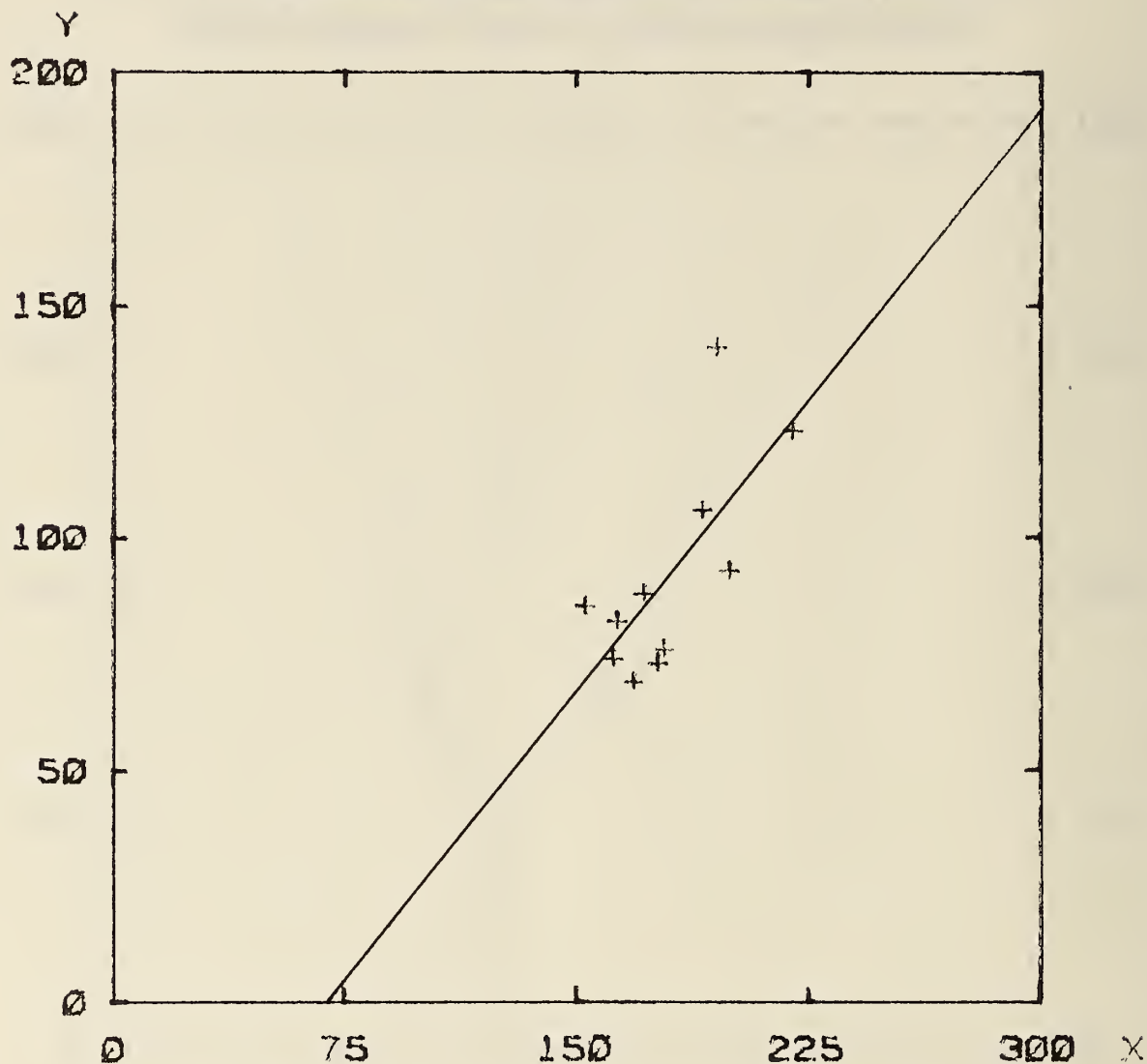
ALUMINUM DEMAND--ABRASIVES THOUSAND SHORT TONS (1960-1976)



PROJECTIONS	Y	X
1985	152.0143	234.1000
2000	214.0779	262.5000

FIGURE F-63. - Scatter diagram of aluminum demand for abrasives use and U.S. population.

ALUMINUM DEMAND--ABRASIVES
THOUSAND SHORT TONS (1960-1976)



GROSS PRIVATE DOMESTIC INVESTMENT
BILLION 1973 \$

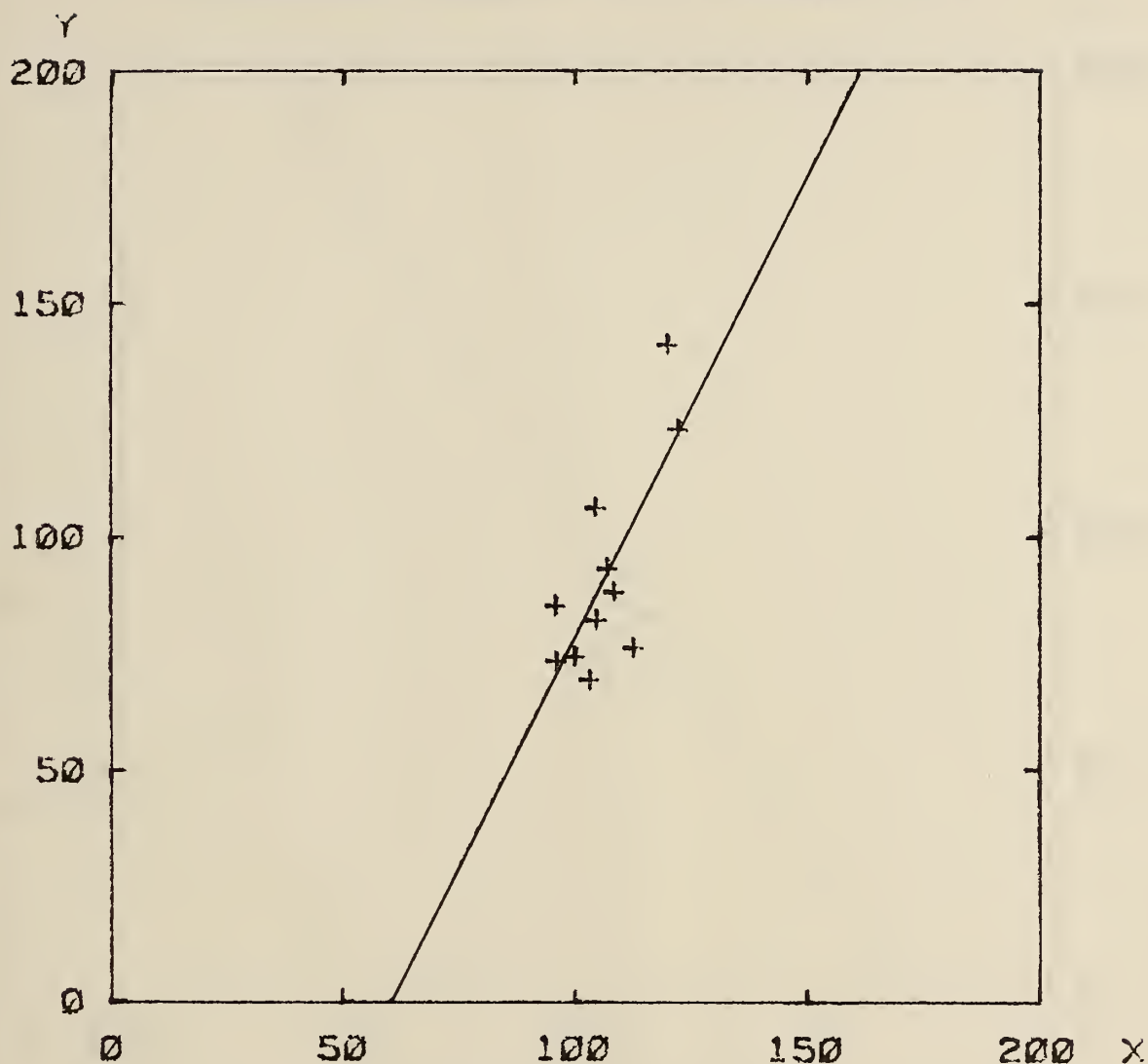
$$Y = -57.7339 + 0.8318 X$$

$$R\text{-SQUARED} = 0.5201$$

PROJECTIONS	Y	X
1985	203.4614	314.0000
2000	374.8189	520.0000

FIGURE F-64. - Scatter diagram of aluminum demand for abrasives use and gross private domestic investment.

ALUMINUM DEMAND--ABRASIVES
THOUSAND SHORT TONS (1966-1976)



FRBI-IRON AND STEEL
1967 = 100

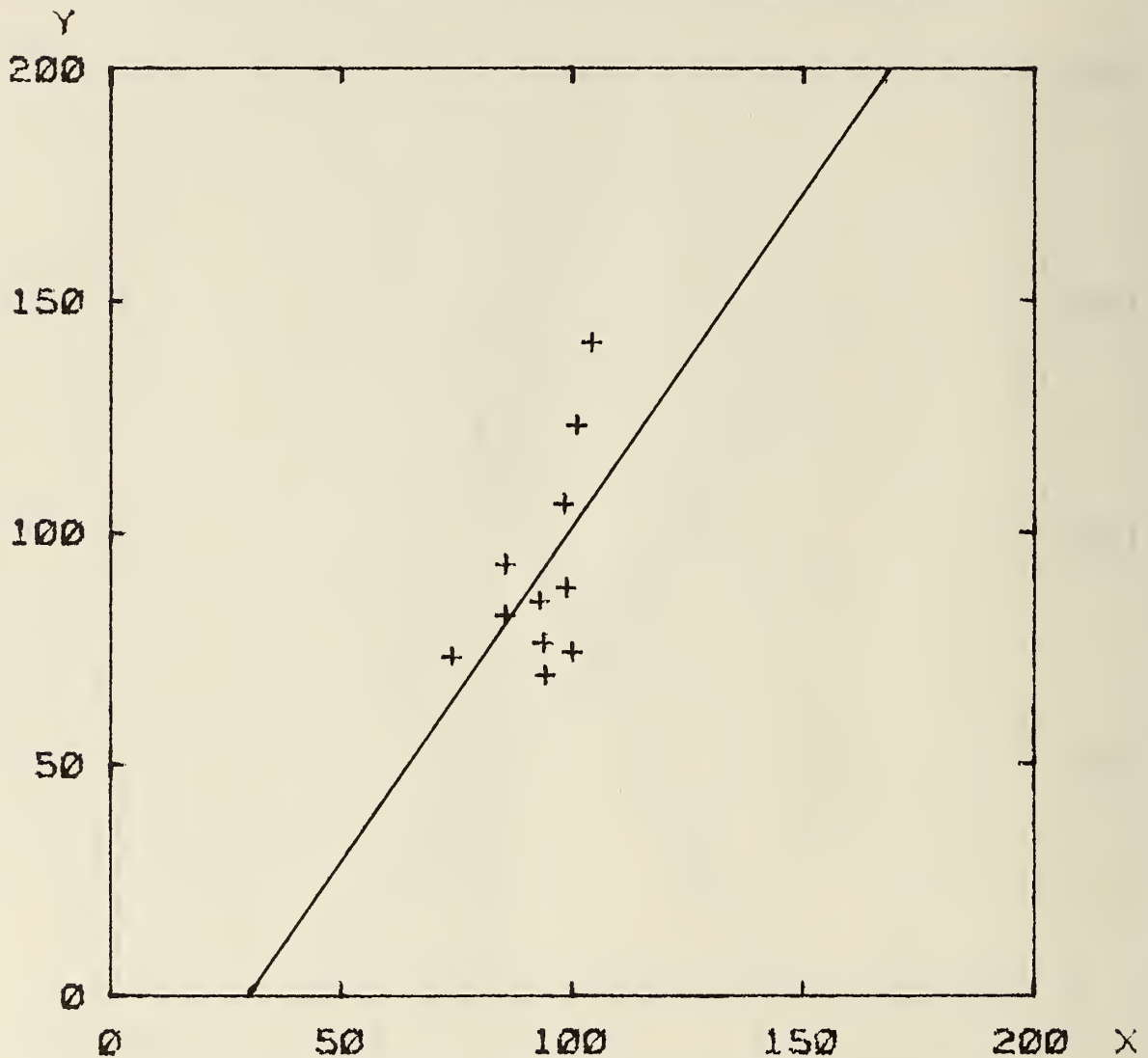
$$Y = -120.3796 + 1.9875 X$$

$$R\text{-SQUARED} = 0.5707$$

PROJECTIONS	Y	X
1985	190.6716	156.5000
2000	285.0801	204.0000

FIGURE F-65. - Scatter diagram of aluminum demand for abrasives use and FRB index for iron and steel.

ALUMINUM DEMAND--ABRASIVES
THOUSAND SHORT TONS (1966-1976)



FRBI-METALWORKING MACHINERY
1967 = 100

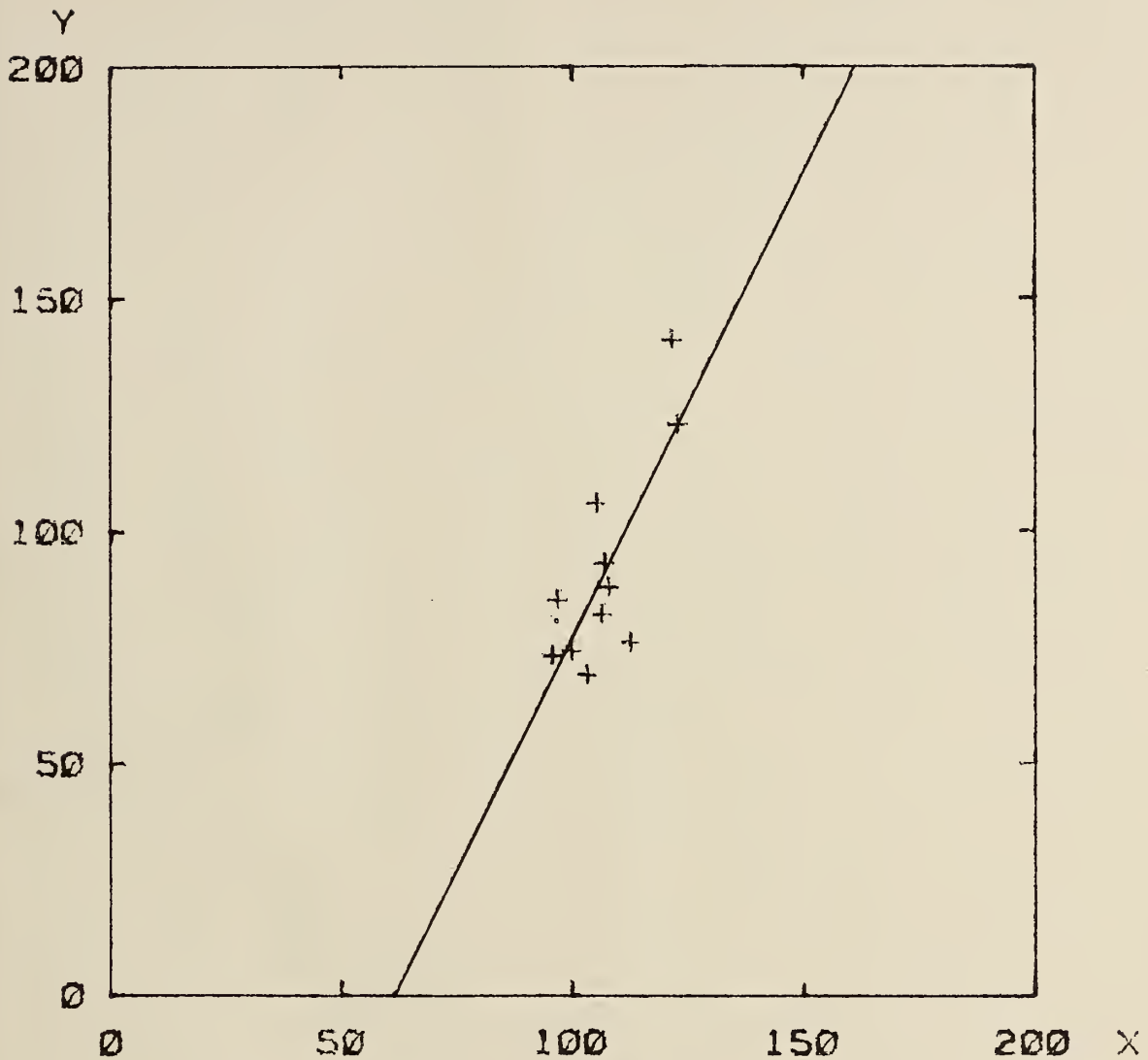
$$Y = -43.3244 + 1.4448 X$$

$$R\text{-SQUARED} = 0.3077$$

PROJECTIONS	Y	X
1985	183.2223	156.8000
2000	284.0703	226.6000

FIGURE F-66. - Scatter diagram of aluminum demand for abrasives use and FRB index for metalworking machinery.

ALUMINUM DEMAND--ABRASIVES
THOUSAND SHORT TONS (1966-1976)



FRBI-BASIC STEEL AND MILL PRODUCTS
1967 = 100

$$Y = -122.8361 + 2.0008 X$$

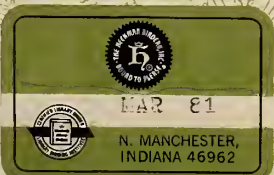
$$R\text{-SQUARED} = 0.6114$$

PROJECTIONS	Y	X
1985	192.8972	157.8000
2000	299.1421	210.9000

FIGURE F-67. - Scatter diagram of aluminum demand for abrasives use and FRB index for basic steel and mill products.

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